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Fragmentation and Strategic Market-Making [1]

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Abstract

Information technology, infrastructure enhancement, and arbitrage strategies all contribute to link trading venues in fragmented markets. Our paper highlights a new cross-market linking channel: the interdependence of liquidity providers' inventory costs. We use a two-venue duopoly model involving strategic risk-averse market-makers. Costs to provide immediacy depend on market-makers' inventory aggregated across venues, implying that absorbing a shock in one venue simultaneously changes marginal costs in all other venues. Moreover, market-makers strategically choose which shock(s) to absorb. These two forces may lead to competitive prices and enhanced liquidity. Using Euronext proprietary data, we uncover evidence for these cross-market inventory cost linkages.

Keywords: Market fragmentation, strategic price competition, cross-market cost linkage

JEL Classification code: D43, G10, G20

1 Introduction

During the past decade major changes in regulation were enacted in the U.S. and in Europe with the intent to promote competition among trading venues. As a result, today's financial markets are more fragmented and more complex than ever. Market fragmentation has multiplied the possibilities of trading the same asset simultaneously across (very) different trading venues, increasing the risk of price discrepancies. New intermediaries have emerged in the form of high-frequency traders, which invest in high speed computerized trading systems to provide liquidity on a given venue and across venues (Menkveld, 2013). As the extent of market fragmentation steadily and quickly increased, cross-market linking strategies have also developed at the same pace.[1] The literature has pointed out arbitrage strategies, duplicate strategies, or directional trading strategies as mechanisms that explain connectedness across venues.[2]

The present paper explores a new and additional channel by which venues are interconnected: the cross-market inventory cost linkage. In our setting, market-making intermediaries trade in multiple venues and exhibit asymmetric costs to supply liquidity due to different positions in the risky asset held in inventory. If a market-maker's costs to provide immediacy are such that a shock absorbed in one venue simultaneously changes her marginal costs in all other venues, then this market-maker updates her quotes across all venues accordingly. We develop this intuition in a two-venue duopoly model in which market-makers' cost to supply liquidity in one venue is linked to all other venues through their inventory position aggregated over all trades. We show that fragmentation may lead to more competition and more liquidity. We test this new result using a proprietary dataset from Euronext on multi-traded stocks, in which we can uniquely identify financial institutions. The granularity of our data allows us to construct the trade-by-trade position of each trader in each venue, and, in particular, the aggregate position of traders engaged in multi-venue market-making. We can thus relate directly aggregate inventory

[1]For instance, Virtu Financial, one of the largest computerized trading firms, trades U.S. securities in a broad set of trading platforms which include NYSE, ARCA, NASDAQ, BATS-Z, BATS-Y, EDGA, or LIGHTPOOL.

[2]See, for instance, the ESMA (2016) report on multi-venue trading strategies.

positions and liquidity and test our main prediction.

Our modeling framework considers two risk-averse market-makers who differ by their inventory position, or, equivalently, by their costs to provide immediacy. The market-maker with the more extreme inventory position can produce immediacy at a smaller cost. Market-makers compete to post prices for the same asset traded on two transparent venues.[3] We assume that the venue termed as the dominant market receives a larger shock than the alternative venue termed as the satellite market. This two-venues environment particularly fits Australian, Canadian or European equity markets in which each incumbent exchange has still a strong presence in its domestic market, facing mainly a strong competitor (Chi-X Australia, Chi-X Canada and CBOE Europe Equities).[4]

The two venues may be exogenously hit by liquidity shocks, which might be of the same sign or of opposite signs. Investigating a two-sided model enables us to pin down the entire mechanics of the cross-market cost linkage, and the strategic behavior of market-makers in each case. Intuitively, when shocks have the same sign, the cross-market cost linkage exerts an anticompetitive force on prices. A market-maker which strategically chooses to absorb a shock in a venue anticipates that her cost to simultaneously provide same-side immediacy increases in the other venue. This effect reverses when shocks have opposite signs: absorbing a buy shock in one venue decreases cost to absorb a sell shock hitting another venue, leading to more competitive prices.

A multi-venue environment generates another force: the possibility to choose the shock to absorb, and thus the venue on which to compete. This force affects both market-makers in our model, but in distinct ways. The market-maker with the smaller costs to supply liquidity can choose to absorb both demand shocks, if her costs are small enough; otherwise, she will choose to absorb the shock with the most favorable impact on her inventory exposure. The second market-

[3]Our paper only focuses on fragmentation of lit (or transparent) trading venues. See, for instance, Buti et al. (2017) for an analysis of market fragmentation involving opaque trading venues (like dark pools).

[4]Chi-X Europe has been acquired by BATS Europe in 2011, which in turn has been acquired by the CBOE in 2017.

maker, who holds a less extreme inventory position, is not able to undercut simultaneously in both venues and will never trade in both. He keeps however the freedom to compete in any of the two venues, thereby endogenously affecting the optimal pricing of the first market-maker.

We show that these two forces, the cross-market cost linkage and the choice to trade in only one of the venues, interact to generate two alternative situations. The first situation consists of a "low-competition" case in which the costs of the more extreme market-maker, even if smaller, are too high to enable her to simultaneously absorb two shocks. She chooses to supply liquidity in the dominant venue while letting her opponent with higher cost absorb the smaller shock of the satellite venue. Each market-maker specializes in one venue and behaves as a local monopolist by pricing high. Even if the cross-market cost linkage may exert a competitive force on prices (occurring when shocks have opposite signs), it is offset by the anticompetitive role played by the possibility for market-makers to compete in only one venue.

The second situation corresponds to an "intense-competition" case in which the inventory costs of the more extreme market-maker are so small that she can absorb all shocks. The intensity of price competition varies however whether liquidity shocks have the same sign or opposite signs. In the latter case, the market-maker uses the competitive role played by the cross-market linkage to post attractive prices in the two venues. In the opposite case, the market-maker must overcome the anticompetitive pressure of the cross-market linkage to undercut her competitor. Additionally, the latter can post aggressive quotes in any of the two venues, which exerts a competitive pressure. These two effects lead to an "ultra-competitive" case, in which the more extreme market-maker prices low in each venue in order to undercut and avoid being undercut. This situation corresponds to the case in which the inventory position among market-makers is highly divergent, and same-sign shocks, if absorbed, enable the more extreme market-maker to reduce her large inventory risk exposure.

Strategic price competition impacts liquidity. We show that the liquidity in each venue varies with the cross-market cost linkage: local liquidity may deteriorate or improve depending on the sign and magnitude of the cross-market cost linkage. By analysing the total liquidity available on the two venues, we find that a fragmented market may be more liquid than a centralized batch market. The improvement in global liquidity is mainly explained by the ultra-competitive case uncovered by our model. Finally the mechanics of the cross-market cross linkage makes liquidity across venues interconnected even if liquidity demands are exogenously specified. This liquidity interconnectedness is stronger when the probability of having same-sign shocks increases, or when volatility or market-makers' risk aversion increases, which might occur in periods of market distress. Our paper therefore proposes thus a new and additional explanation related to commonality in liquidity.

To test the model, we adopt a two-step empirical approach. In the first step, we investigate whether intermediaries' costs to supply liquidity are interrelated across venues. In the second step, we test the main prediction of our model, i.e., that bid-ask spreads within one venue vary with the sign of the shock in the other venue (identical or opposite) and with the divergence in market-makers' aggregate inventory positions. The more divergent market-makers' aggregate inventories are, the more likely a more extreme market-maker is willing to post ultra-competitive quotes.

Our analysis uses a proprietary dataset on multi-venue traded stocks from Euronext on a four-month period in 2007. This environment provides an excellent laboratory to test our main prediction for three reasons. First, within Euronext created in 2000, trading rules in all markets (Amsterdam, Brussels, Paris and Lisbon) are harmonized and structured on the Paris Bourse limit order book model. During our time period, limit order books are identical but separate. Second, during that period (that is, before the implementation of MiFID in November 2007), Euronext collected the overwhelming majority of the trades (up to 98 %).[5] Our reconstitution of intermediaries' net positions across venues is therefore a very good proxy for their aggregate inventory position. Third, the proliferation of trading venues post-MiFID is associated with

[5] The European MiFID legislation (Markets in Financial Instruments Directive) has put an end to the monopoly of regulated markets with the introduction of multilateral trading facilities.

platforms with different trading fees, clearing and settlement systems, tick sizes, speed (or latency), or degree of transparency (like dark pools) which all affect bid-ask spreads and market quality.[6] In other words, the effects of fragmentation might be confounded by these different forces. Our study does not suffer from this issue and clearly focuses on the unique impact of market fragmentation on price competition and liquidity.

A strength of our dataset is to contain the unique identifier of each participant (identical across all venues), enabling us to track members' activity (trades and messages) from one venue to another. Figure 1 illustrates our data. The top graph shows the multi-venue quoting activity of a Euronext market-maker trading the French gas utility Suez both on Euronext Paris and Euronext Brussels on January 19, 2007. The bottom graph shows the aggregate inventory position. Interestingly this inventory tends to mean-revert over the day, corroborating that multi-venue market-makers manage inventory across all venues. Her quoting competitiveness also varies across hours and across venues (quotes of the satellite venue are more distant from the midpoint of the virtual consolidated book).

[INSERT FIGURE 1]

In accordance with Figure 1, our empirical analysis finds evidence of cross-venue inventory effects. Using a logit model, we find that multi-venue market participants, in particular formally registered market-makers, are more likely to submit messages aiming at mean-reverting inventory in a venue when their preexisting orders have been passively hit in the other venue. This result validates our hypothesis that aggregate inventory is a driver of multi-venue market-making strategies. It also makes this paper one of the first to uncover evidence on cross-venue inventory effects. More importantly, our empirical analysis shows that bid-ask spreads decrease when our measure of divergence in inventories is high and when liquidity shocks across venues have the same sign, i.e. when competition is heating up. This result is uniquely predicted by our model and it is the opposite of what an adverse-selection-based model would predict.

[6] See Gomber et al. (2017) for a survey on the impact of market fragmentation.

The negative impact of fragmentation may be overcome by linking venues, as suggested by Foucault et al. (2013). O'Hara and Ye (2011) hypothesize that a consolidated tape and a trade-through protection are necessary conditions to create a virtual consolidated market. The use of Smart Order Routing Technologies (SORT) by market participants has contributed to link the venues together and to consolidate the market. Foucault and Menkveld (2008) show that, provided that they have access to a SORT, limit order traders increase competition among venues by bypassing time priority of the dominant venue. This "queue-jumping" strategy results in higher depth and lower spreads in a virtual consolidated book. van Kervel (2015) extends the set-up of Foucault and Menkveld (2008) to informed fast traders, and shows however that an increase in the proportion of fast investors using SORT leads to an increase in adverse selection costs for market-makers leading to lower liquidity. While these papers assume that quotes are set by competitive market makers, we develop a model of strategic market-making and show that the best response of market-makers to non constant marginal costs to supply liquidity across venues may lead to more competition and lower spreads. Our model is consistent with empirical results uncovering positive effects of liquidity of market fragmentation (see, for example, Degryse et al. (2015) or Aitken et al. (2017)), while offering a new and alternative mechanism linking venues.

The paper is organized as follows. Section 2 describes the model and investigates price formation in a two-venue market-making environment. Section 3 describes the data, provides summary statistics and tests the main implications of the model. Section 4 concludes the paper. All proofs are available in the Appendix.

2 The Model

2.1 The basic setting

We consider the market for a risky asset with a random final cash-flow $\tilde{v}$ which is normally distributed with expected value μ and variance σ^2. There are two types of market participants: investors who demand liquidity and market-makers who supply liquidity.

Time line of the trading process. The trading game consists of four stages. At stage 1, market-makers are endowed with an inventory position. At stage 2, a venue can be hit by an exogenous liquidity shock generating a liquidity demand Q. At stage 3 market-makers compete to execute the demand. At stage 4, the final cash-flow of the risky asset is realized.

Stage 1 - Reservation prices and costs to supply liquidity. Liquidity is supplied by two equally risk-averse intermediaries with coefficient ρ.[7] At stage 1, each market-maker i receives a non-zero inventory position in the risky asset I_i, where I_i is the realization of the random variable $\tilde{I}_i$ uniformly distributed on $[I_d, I_u]$ $(i = 1, 2)$.[8] We denote r_i the minimum (resp. maximum) selling (resp. buying) price at which market-makers can execute buy orders (resp. sell orders) without incurring losses. The reservation price r_i is defined by equating expected utility functions $EU(Q, r_i) = EU(0, r_i)$, where $U(Q, r_i)$ is the (CARA) utility of market-maker i absorbing the demand shock Q at price r_i, i.e.,

$$r(Q; I_i) \equiv r_i(Q) = \mu - \rho\sigma^2 I_i + \frac{\rho\sigma^2}{2}Q. \tag{1}$$

Because of the non-zero inventory, market-makers are willing to trade to reduce inventory risk. Equation (1) shows an inverse relationship between a market-maker's inventory position and her/his reservation prices. Longer market-makers with lower reservation prices are actually induced to post lower bid and ask prices to attract buy orders and reduce their inventory

[7] In the remaining part of the paper, we use market-maker, intermediary or liquidity provider interchangeably.

[8] All random variables are independent and their distributions are common knowledge.

exposure. However, the last term shows that, the larger the shock Q to absorb, the higher the quote a market-maker would post, making him less likely to attract buy orders.

Reservation prices may be interpreted as costs to supply liquidity for market-makers, which allows us to define (total) inventory costs for market-maker i as follows:

$$TC_i(Q) = r_i(Q) \times Q, (i = 1, 2). \tag{2}$$

Note that inventory costs are quadratic.[9] Large transactions are more risky and thus more costly as they may lead to more unbalanced inventory positions for market-makers.

For ease of exposition, in what follows we consider that market-maker 1 is endowed with a longer inventory position, i.e., $I_1 > I_2$. This assumption entails that costs to absorb the shock Q are smaller for market-maker 1: $TC_1(Q) < TC_2(Q)$.

Stage 2 - Market fragmentation. We suppose that the risky security trades in two trading venues, denoted D and S, that we assume to be transparent. At stage 2, a venue m can be exogenously hit by a liquidity shock with probability ζ_m ($m = D$, or S).[10] We assume that the liquidity demand sent to venue D, denoted Q_D, is larger in magnitude than that routed to venue S, i.e., $|Q_D| > |Q_S|$ and we assume that probabilities of shocks are such that $\zeta_D > \zeta_S$. We thus term venue D as the dominant market, and venue S as the satellite market. Note that, by convention, a positive (resp. negative) shock generates a buy (resp. sell) liquidity demand denoted $Q_m > 0$ (resp. $Q_m < 0$), $m = D$, or S. We denote by ζ the probability that both venues are simultaneously hit by shocks ($\zeta = \zeta_D \times \zeta_S$), and by γ the probability that shocks have the same sign across venues.

For the sake of exposition we assume that shocks generate a net-buying order flow, i.e.,

[9]This assumption is common in the theoretical literature modelling risk-averse market-makers: see, for instance, Ho and Stoll (1983), Biais et al. (1998), or Gârleanu and Pedersen (2013).

[10]The baseline model assumes that the order flow exogenously fragments across venues D and S. In the Online Appendix (Appendix C), we address the case of an endogenous fragmentation by assuming that a global liquidity demander has simultaneously access to all venues and minimizes his trading costs by optimally splitting orders across venues. Appendix C shows that, even in this case, the liquidity demander does not direct all orders to a unique venue, but optimally chooses to split orders, leading to a fragmented market.

$Q_D + Q_S > 0$, or, equivalently, $Q_D > 0$ while Q_S might be a buying or selling liquidity demand. Symmetric results are easily obtained for a net-selling order flow. Note that, since the global order flow is net-buying ($Q_D + Q_S > 0$), market-maker 1 benefits from a competitive advantage (given that we assume $I_1 > I_2$, or, equivalently $TC_1(Q_D + Q_S) < TC_2(Q_D + Q_S)$).

Stage 3 - Quoting strategies of multi-venue intermediaries. We assume that intermediaries behave strategically and that they have access to all trading venues at the same time. At stage 3, conditional on observing Q_D and Q_S, multi-venue market-makers post *simultaneously* their quotes in venues D and S. The market-maker who posts the lowest ask price (resp. highest bid price) in venue m executes $Q_m > 0$ (resp. $Q_m < 0$), for $m = D, S$.

A multi-venue quoting strategy for market-maker i is a pair of quoted prices (p_i^D, p_i^S) where p_i^D is the price posted by market-maker i in venue D and p_i^S is the price posted by i in venue S (which is an ask price if $Q_m > 0$ or a bid price if $Q_m < 0$). Market-makers' trading profits are detailed in Appendix A.1.

In our model, we will need to consider under what conditions a market-maker who competes in one venue will decide to compete in an additional venue. Denote by Q_{-m} the liquidity shock in the additional venue, given that the market-maker is ready to absorb the shock Q_m in his "home" venue. We introduce a specific reservation price $\hat{r}_i$, termed as the "stay-at-home" price, at which market-maker i is indifferent to execute the liquidity demand Q_{-m} in addition to the demand Q_m. Specifically let $\hat{r}_i(Q_{-m})$ be defined by equating $EU(Q_{-m}+Q_m, \hat{r}_i) = EU(Q_m, \hat{r}_i)$. It follows that:

$$\hat{r}_i(Q_{-m}) = \mu - \rho\sigma^2 I_i + \frac{\rho\sigma^2}{2} Q_{-m} + \rho\sigma^2 Q_m = r_i(Q_{-m}) + \rho\sigma^2 Q_m. \tag{3}$$

We observe that the "stay-at-home" price may be rewritten as $\hat{r}_i(Q_{-m}) = r(Q_{-m}; I_i - Q_m)$. In other words, market-maker i behaves as if she is sure to execute (inelastic) orders in venue m (consistent with a monopolistic situation) and, anticipating the impact of Q_m on her inventory

$(I_i - Q_m)$, her true value for accepting to enter in the other venue $-m$ is now this new reservation price, $\hat{r}_i(Q_{-m})$. For example, if $Q_m > 0$, any selling price below $\hat{r}_i$ is not sufficiently high for the market-maker to try to capture the orders Q_{-m}. She prefers not to compete in the additional venue. Notice that $\hat{r}_i(Q_{-m}) > r_i(Q_m + Q_{-m})$ if $Q_m > 0$ and $\hat{r}_i(Q_{-m}) \leq r_i(Q_m + Q_{-m})$ if $Q_m \leq 0$.

Stage 4 - End of the trading game. Because the final cash-flow of the risky asset is realized, no uncertainty remains. The extensive form of the trading game is represented in Figure 2.

Before we leave this section, two important remarks are in order. First, in our set-up market-makers must manage their inventory by keeping track of shocks across all trading venues. Because making the market "globally" (i.e., across various venues) affects an intermediary's total exposure to inventory risk, only aggregate inventory matters as opposed to ordinary inventory that guides an intermediary taking risks just in one venue.[11] Second, because we assume that venues D and S are transparent, intermediaries' quotes are observable by market participants.[12] In other words, we suppose that intermediaries observe each other's inventory position, or, equivalently, each other's private costs to provide liquidity.

2.2 Equilibrium quotes in a fragmented market

This section analyzes the Nash equilibria of the quoting game.

Let us first consider a centralized market in which liquidity demands are batched and sent to a unique venue, as in Ho and Stoll (1983). In this case the market-maker with a longer inventory position (market-maker 1 by assumption) posts a more competitive ask price, by slightly undercutting the reservation price of her shorter opponent:

$$(a_1^c)^* = r_2(Q_D + Q_S) - \varepsilon, \tag{4}$$

[11]Our definition of aggregate inventory is close to the definition of *equivalent* or total inventory emphasized by Ho and Stoll (1983) and discussed in Naik and Yadav (2003). However, while equivalent inventory is the overall position of an intermediary across all stocks, aggregate inventory is the cumulated net inventory position of an intermediary in a single stock but across all available trading venues.

[12]The game is a complete information game.

where ε corresponds to one tick. The longer intermediary behaves strategically by shading her ask price upward, i.e. she chooses to post an ask price above her reservation price $(r_1(Q_D+Q_S))$ to increase her payoff, but still below the true value of her opponent to be sure to execute the order flow. This section analyzes how market fragmentation significantly alters this market-making strategy, and market-maker 1's market power.

2.2.1 Preliminary results

When markets are fragmented, market-makers meet and strategically compete in many different venues, and not in a single one. One consequence of this multi-market structure is that they might strategically choose to withdraw from some venues to compete more intensely on other venues. In a two-venue setting, Lemma 1 below shows that, at equilibrium (if it exists) two different situations might emerge: either a single market-maker virtually consolidates the market by executing all orders across venues, or each market-maker specializes in one venue, by trading only the orders from that venue.

Lemma 1 *Assume that $I_1 > I_2$ and that $Q_D + Q_S > 0$.*

1. If market-makers' inventory costs are such that $TC_1(Q_D + Q_S) < TC_1(Q_D) + TC_2(Q_S)$ or, equivalently, $(I_1 - I_2 - Q_D)Q_S > 0$, and if an equilibrium exists, then a market-maker consolidates the order flow through a multi-venue execution. Conversely, if $TC_1(Q_D + Q_S) \geq TC_1(Q_D) + TC_2(Q_S)$ or, equivalently, $(I_1 - I_2 - Q_D)Q_S \leq 0$, and if an equilibrium exists, then it is such that orders submitted to the different venues are executed by different intermediaries.

2. If there exists an equilibrium such that a market-maker consolidates the orders, then the longer market-maker executes the global order flow. If there exists an equilibrium such that each market-maker specializes in one venue, then the longer market-maker executes the buy demand sent to the dominant venue, while the shorter intermediary executes orders sent to the satellite venue.

In a centralized market, recall that orders are batched and crossed (if $Q_S < 0$) and the outcome depends only on the divergence between market-makers' inventory positions $(I_1 - I_2)$ because the latter determines who has the lowest cost to supply liquidity. In a two-venue setting, the problem is more complex. First, in order to decide which shock(s) to absorb, market-makers have to consider the different costs to supply liquidity in each venue, $TC_i(Q_m)$, and also across venues, $TC_i(Q_m + Q_{-m})$, as shown in Lemma 1.

Second, Lemma 1 indicates that there exist cases in which market-maker 1 behaves as if capacity-constrained. Even if market-maker 1 has the smallest cost to produce liquidity, this cost may however be too high to profitably absorb all shocks, as indicated by the inequality $TC_1(Q_D + Q_S) \geq TC_1(Q_D) + TC_2(Q_S)$.[13] In that case market-maker 1 executes only orders in the venue with the most favorable impact on her inventory risk, that is, venue D. For instance, consider the case in which the divergence in inventories is high, i.e., $I_1 - I_2 - Q_D \geq 0$, and the shock hitting S is negative ($Q_S < 0$). Since market-maker 1's position is very large and very risky, she is willing to execute all incoming *buy* orders to lay off her inventory. Hence she trades only Q_D. Absorbing sell orders in S would indeed aggravate her inventory exposure. In a two-venue setting, there exists a possibility to compete in only one venue, which, in turn, influences market-makers' pricing strategies.

Third, recall that market-makers' inventory position is aggregated across venues. This global inventory position makes market-makers' costs to supply liquidity interdependent across venues. The marginal cost to supply liquidity in venue m depends also on the *output* in venue $-m$: $\frac{\partial TC_i(Q_m+Q_{-m})}{\partial Q_m} = \frac{\partial TC_i(Q_m)}{\partial Q_m} + \rho\sigma^2 Q_{-m}$. The second term, $\rho\sigma^2 Q_{-m}$, is a new cross-market effect, absent from any competition in a single centralized venue, which influences the strategic pricing decision of market-makers. If market-maker i chooses to absorb a buy (resp. sell) demand in venue $-m$, her cost to provide liquidity in m increases (resp. decreases), resulting in higher (resp. lower) selling prices in m. This pattern is perfectly anticipated by her opponent in

[13] In our model, inventory position plays a role of a "soft" capacity constraint, because market-makers can always absorb larger trades beyond their optimal inventory position, albeit at an increasing marginal cost. The softness of capacity constraints has been introduced by Cabon-Dhersin and Drouhin (2017).

our model. The cross-market cost linkage, created by the existence of a second venue, may thus affect competition either way - by softening it (when shocks hitting venues have the same sign) or intensifying it (when shocks have opposite signs). Notice that the anticompetitive vs. competitive effect of the cross-market cost linkage on price competition is caused by both non-constant marginal costs to supply liquidity and the strategic behavior of market-makers.

In summary, a two-market structure involves new strategic interactions across venues, namely through the structure of interdependent inventory costs and the possibility to choose the shock to absorb. These interactions fundamentally change the outcome of price competition, as shown in Proposition 1 below.

2.2.2 Optimal quotes

We now analyse the pricing strategies of market-makers.

Proposition 1 *Assume that $I_1 > I_2$ and $Q_D + Q_S > 0$.*

1. If $(I_1 - I_2 - Q_D)Q_S > 0$, there exists a Nash equilibrium, in which market-maker 1, with the longer position, consolidates the market by posting the best prices in all venues. At equilibrium,

$$\begin{cases} \left((a_1^D)^*, \quad (a_1^S)^*\right) = \left(r_2(Q_D) - \varepsilon, \quad r_2(Q_S) - \varepsilon\right) & \text{if } Q_S > 0, \\ \left((a_1^D)^*, \quad (b_1^S)^*\right) = \left(\hat{r}_2(Q_D) - \varepsilon, \quad r_2(Q_S) + \varepsilon\right) & \text{if } Q_S < 0. \end{cases}$$

2. If $(I_1 - I_2 - Q_D)Q_S \leq 0$, there exists a unique Nash equilibrium, in which market-maker 1, holding the larger inventory, posts the best selling price in the dominant market while market-maker 2 posts the best price in the satellite market, that is:

$$\begin{cases} \left((a_1^D)^*, \quad (a_2^S)^*\right) = \left(\hat{r}_2(Q_D) - \rho\sigma^2 Q_S \times \eta - \varepsilon, \quad \hat{r}_1(Q_S) - \varepsilon\right) & \text{if } Q_S > 0, \\ \left(((a_1^D))^*, \quad (b_2^S)^*\right) = \left(\hat{r}_2(Q_D) - \varepsilon, \quad \hat{r}_1(Q_S) + \varepsilon\right) & \text{if } Q_S < 0. \end{cases}$$

where ε corresponds to the minimum tick size and η is equal to $\frac{(I_1 - I_2)}{Q_D} \in]0, 1]$.

Recall that there are two driving forces specific to our two-market duopoly: (i) the possibility to choose the venue on which to compete; and (ii) a cross-market cost linkage due to the global management of the position across venues. The net effect of these forces creates two opposite situations in our model: (i) an "intense-competition" case in which costs to supply liquidity are small enough to allow market-maker 1 to price low in the two venues in order to undercut and to avoid being undercut ($TC_1(Q_D + Q_S) < TC_1(Q_D) + TC_2(Q_S)$), and (ii) a "low competition" case in which market-maker 1 cannot absorb shocks in the two venues and prices high to maximize profit in one venue while market-maker 2 chooses the other venue ($TC_1(Q_D + Q_S) \geq TC_1(Q_D) + TC_2(Q_S)$).[14]

First, let us consider the "intense-competition" case, in which market-maker 1' inventory costs are small enough to provide liquidity in the two venues (that is, $(I_1 - I_2 - Q_D) \times Q_S > 0$ holds).

- If $Q_S > 0$ and market-maker 1 is very long ($I_1 - I_2 - Q_D > 0$), market-maker 1 has incentives to undercut market-maker 2 in both venues. Her opponent however might choose to compete in a single venue, which might be either D or S. Market-maker 1 is thus obliged to quote below the minimum selling price of market-maker 2, $r_2(Q_m)$, in each venue m. The threat created by the possibility of the opponent to compete in only one venue forces market-maker 1 to quote "ultra-competitive" prices. This pressure offsets the anticompetitive role of the cross-market cost linkage.

- Suppose that $Q_S < 0$ and market maker 1 is less long ($I_1 - I_2 - Q_D < 0$). If market-maker 1 executes buy orders sent to D, she will be shorter than market-maker 2. She is thus able to undercut market-maker 2's highest possible buying price, $r_2(Q_S)$ in venue S. Further the cross-market cost linkage plays a competitive role. In particular, it allows market maker 1 to decrease even more her selling price to $\hat{r_2}(Q_D) < r_2(Q_D)$ in venue D.

[14] In a single centralized venue, one can also show that a market-maker with more severe capacity constraints posts less competitive prices, and enjoys a higher market power, while market-makers with large capacity behave more competitively.

Note that the competitive effort exerted by market-maker 1 is more intense when $Q_S > 0$. Due to the inventory cost convexity, supplying liquidity for same-sign shocks is much more costly than for opposite-sign shocks (ceteris paribus), which is mirrored by the positive or negative impact of the cross-market cost linkage. That is the reason why we qualify quotes posted by the longer market-maker as ultra-competitive.

Second, let us consider the "low competition" case in which market-maker 1's inventory costs are too high to supply liquidity in the two venues, which is equivalent to the following inequality $(I_1 - I_2 - Q_D) \times Q_S \leq 0$.

- Suppose that $Q_S > 0$ and that market-maker 1 is less long, $I_1 - I_2 - Q_D \leq 0$. If she is in position to undercut market-maker 2 in the dominant venue D, she is not long enough to undercut in venue S ($I_1 - Q_D \leq I_2$). She thus chooses to quote her "stay-at-home" price in S, which is anticipated by marker-maker 2. To be sure to undercut and not being undercut in the dominant venue, market-maker 1 must however quote a price more aggressive than the "stay-at-home" price of market-maker 2, resulting in posting the price $(a_1^D)^* \leq \hat{r}_2(Q_D)$. Both the cross-market cost linkage and the possibility to absorb only orders sent to one's "home" venue play an anti-competitive role, resulting in less competitive prices.

- Suppose now that $Q_S < 0$ and that market-maker 1 is very long ($I_1 - I_1 \geq Q_D$), exposing her to large inventory risks. Reducing her inventory imbalance is the primary consideration for the market-maker choice of trading venue. She does choose to compete in venue D and not in venue S. Note that even if she undercuts market-maker 2 in venue D, she is still longer than him ($I_1 - Q_D \geq I_2$) and has no chance to execute sell orders in venue S. Symmetrically the shorter market-maker 2 chooses to compete in venue S, and not in venue D. This creates a situation in which each market-maker acts as a monopolist in its preferred or "home" venue and quotes in the other venue her/his "stay-at-home" price $\hat{r}_i(Q_{-m})$. Even if the cross-market cost linkage exerts a competitive force on prices, it is more than offset by the anti-competitive role of the ability to choose the shock to absorb.

2.2.3 Transaction costs

Based on Proposition 1 this section investigates transaction costs by comparing the level of best bid and ask prices set in a fragmented market to those in a centralized market. Liquidity traders face lower total trading costs (denoted $TTrC$) in a fragmented market if and only if:

$$TTrC - TTrC^c = (a_1^D)^* Q_D + (p_i^S)^* Q_S - (a_1^c)^*(Q_D + Q_S) \leq 0 \tag{5}$$

To make the analysis easier, we use a numerical example. Figure 3 shows the best prices as a function of the divergence in inventories $(I_1 - I_2)$. Panel A illustrates the case in which two positive shocks simultaneously hit venue D and venue S. Panel B illustrates the case of shocks of opposite signs. In both cases, the vertical line Q_D separates the region in which the divergence in intermediaries' inventories is high (the right-hand side of the graph, corresponding to $I_1 - I_2 > Q_D$) from the region in which divergence is low (the left-hand side, corresponding to $I_1 - I_2 \leq Q_D$).

[INSERT FIGURE 3]

The case of simultaneous positive shocks. In this case, the cross-market cost linkage has an anticompetitive impact on the best prices in both venues D and S. However, the possibility to compete in only one venue sometimes offsets this negative force: In the region to the right of the vertical line Q_D, market-maker 1 is very long and posts ultra-competitive prices in each venue to be sure to undercut market-maker 2. In this case, market fragmentation increases intra-venue competition leading to lower transaction costs relative to the centralized case: $TTrC - TTrC^c = (a_1^D)^* Q_D + (a_1^S)^* Q_S - (a_1^c)^*(Q_D + Q_S) = -\rho\sigma^2 Q_D Q_S < 0$.

In the region to the left of the vertical line Q_D, it is more profitable for market-maker 1 to supply liquidity in only one venue. She chooses to absorb the shock hitting the dominant market, while letting her opponent absorb the less desirable shock in the satellite venue. The market is split among the two market-makers.

Interestingly, the equilibrium selling price in the satellite venue might be higher than the one of the dominant venue despite a smaller quantity to execute (thus a smaller price impact). The intuition for this result is as follows. When marker-makers' position tend to be equal ($I_1 - I_2 \to 0$), the longer market-maker still executes the larger demand, while the shorter market-maker executes the smaller demand. A higher equilibrium price in the satellite market must however compensate the smaller quantity executed by market-maker 2 to prevent him from deviating and executing the larger quantity. Therefore there must exist an intersection point p at which selling prices are equal across venues, as illustrated by Figure 3 Panel A.

Market fragmentation has thus an ambiguous effect on price competition and transaction costs in this case. To the right of p, price competition is still stronger than in the centralized case and market fragmentation is beneficial for transaction costs ($TTrC - TTrC^c = -2\rho\sigma^2 \times Q_S(I_1 - I_2 - \frac{Q_D}{2}) \leq 0$). To the left of p, competition is weaker, and transaction costs are larger than those paid in a centralized market: $TTrC - TTrC^c > 0$.[15]

The case of opposite shocks. Subfigure (a) of Panel B depicts the best selling price in the dominant venue. Subfigure (b) draws the best buying price in the satellite venue (which is hit by a sell shock). In this case, remind that the cross-market cost linkage exert a competitive force on quoted prices. In particular, the equilibrium selling price in the dominant venue $(a^D)^*$ is always more competitive than the ask price of a centralized market, which is mainly driven by the competitive effect of the cross-market cost linkage.

When market-maker 1 is very long (region to the right of the vertical line Q_D), she chooses not to compete for sell orders sent to the satellite venue, that would increase inventory risk. This is anticipated by market-maker 2 who therefore posts a price which is less and less aggressive than market-maker 1 is longer. Both market-makers behave as local monopolists. Transaction costs thus worsen: $TTrC - TTrC^c = \rho\sigma^2(I_1 - I_2 - Q_D)(-Q_S) \geq 0$.

[15] We can easily deduce that p is such that prices in a fragmented and a centralized market are equal, or such that transaction costs are equal across market structures, i.e., p is such that $I_1 - I_2 - Q_D/2 = 0$.

In the region to the left of the vertical line Q_D, market-maker 1 consolidates the fragmented order flow $Q_D + Q_S$. She undercuts her opponent in venue D, which simultaneously makes her shorter than market-maker 2 and able to also undercut him in the satellite venue. While orders cannot be crossed directly in a two-venue setting, prices posted in each venue by market-maker 1 are however as competitive as in a centralized market crossing orders. Fragmentation is innocuous since $TrC - TrC^c = 0$.

[INSERT FIGURE 4]

Figure 4 summarizes the impact of multi-venue strategic market-making on ex post transaction costs (for any values of our parameters), compared to strategic market-making in a centralized market. The region above the x-axis represents the case in which shocks have the same sign, while the region below is related to the case in which shocks have opposite signs. A global overview of the picture shows that market fragmentation is beneficial when shocks have the same sign (the green regions B and C), as it strengthens market-makers' competition. It is globally the opposite situation when shocks have opposite signs, since transaction costs are, at best, similar to those paid in a centralized market (the neutral white region D), or higher (the red region $A2$). In that case a centralized market is better since it crosses trades and only the net order imbalance ($Q_D + Q_S < Q_D$) is absorbed by the longer market-maker. The consolidation advantage is however replicated in a two-venue setting when the best response to market-maker 1 given her small inventory risk exposure is to virtually consolidate the net order flow (region D of low inventory divergence).

2.3 Assessing ex ante execution quality

This section analyzes how multi-venue market-making strategies affects liquidity. Using the terminology developed in Degryse et al. (2015), we investigate local liquidity by computing expected bid-ask spreads set in each venue, and global liquidity by aggregating expected transaction costs over the two venues.

2.3.1 Local liquidity

Using Proposition 1 and the extensive form of the trading game (Figure 2), we compute the expected (half-) spreads in the dominant and the satellite venues for any set of inventory positions and any sign for liquidity shocks Q_D and Q_S. For ease of exposition, we denote by ϕ_m the magnitude of the shock hitting venue m scaled by the distribution support $(I_u - I_d)$ and signed according to the sign of the shock: $\phi_m = \frac{Q_m}{I_u - I_d}$ for a positive shock and $-\phi_m = \frac{Q_m}{I_u - I_d}$ for a negative shock. Proposition 2 follows.

Proposition 2 *The expected (half-) spreads in the dominant and the satellite venues respectively write:*

$$E\left(s^D\right) = \rho\sigma^2(I_u - I_d)\left[\frac{1}{2}(\phi_D - \frac{2I_d + I_u}{3}) + \zeta_S\phi_S\left[\gamma(\phi_D - \frac{(\phi_D)^2}{3}) - (1-\gamma)\right]\right], \quad (6)$$

$$E\left(s^S\right) = \rho\sigma^2(I_u - I_d)\left[\frac{1}{2}(\phi_S - \frac{2I_d + I_u}{3}) + \zeta_D\phi_D\left[\phi_D - \frac{(\phi_D)^2}{3} - (1-\gamma)\right]\right], \quad (7)$$

where ζ_m is the probability that a liquidity shock hits venue m and γ is the probability that shocks hitting D and to S have the same sign ($m = D, S$).

Local spreads are made of two components. The first component is the *direct* price impact of orders routed to that venue. It corresponds to the expected best offer that would prevail if there is no shock hitting the other venue (ϕ_{-m} is zero with probability $1-\zeta_{-m}$). The second component consists of the *indirect* price impact of trading in the other venue (ϕ_{-m}) resulting from the effect of the cross-market cost linkage, while its magnitude relates to market-makers' market power. This component may be positive or negative depending on the value of the parameters γ and ϕ_D. In particular, local expected spreads adversely enlarge when γ, the probability that shocks have the same signs across venues, increases. Local liquidity deteriorates mainly due to the anticompetitive role of the cross-market cost linkage. When γ is sufficiently low, the opposite occurs due to the changing sign of the cross-market cost linkage.

Note that the existence of a second market is asymmetric: the dominant market has a stronger influence on local spreads set in the satellite market, than the opposite.[16] It is due to two main effects: (i) the magnitude of the indirect shock, bigger from the dominant venue (stronger effect of the cross-market cost linkage); (ii) the intensity of the competition which is lower in the satellite market (see Panel A of Figure 3 to the left of p and Panel B (b) to the right of Q_D).

2.3.2 Global liquidity

From Proposition 2, we compute total expected trading costs in a fragmented market. The next corollary compares them to expected trading costs that would prevail in a centralized market.

Corollary 1 *Total expected trading costs are lower in a fragmented market rather than in a centralized market if and only if the probability to observe shocks with the same sign is such that* $\gamma > \frac{1}{3}$ *and the standardized quantity* ϕ_D *is neither too large, nor too small (*$\Phi^1_\gamma < \phi_D < \Phi^2_\gamma$*).*

The intuition of the corollary is as follows. Figure 4 shows that ex post transaction costs are strictly lower in a fragmented market in two regions B and C, which are such that shocks have the same sign and the divergence in inventories is not too low ($I_1 - I_2 > \frac{1}{2}Q_D$). Therefore, expected transaction costs should be lower in a fragmented market when the probability to observe shocks of the same signs is sufficiently high and the probability of having divergent inventories among market-makers is also sufficiently high. The latter condition depends on ϕ_D, which should not be too large ($\phi_D < \Phi^2_\gamma$) for that condition to be true. When the probability that shocks have opposite sign increases ($\gamma \to 1/3$), prices are at best as competitive as in a centralized market (region D) or worse (region $A2$). For innocuous expected transaction costs, the probability of observing a very low divergence in inventories should be high, i.e., the standardized quantity ϕ_D should not be too small for this probability to be high ($\Phi^1_\gamma < \phi_D$).

[16] Given that $\zeta_D > \zeta_S$, $\phi_D > \phi_S$, and $(1-\gamma)(\phi_D - \frac{(\phi_D)^2}{3}) \geq 0)$, we deduce that $\phi_D - \frac{(\phi_D)^2}{3} - (1-\gamma) > \gamma(\phi_D - \frac{(\phi_D)^2}{3}) - (1-\gamma)$. The indirect impact of a second venue is thus stronger for a satellite venue.

Propositions 2 and Corollary 1 imply that when same-sign shocks are more likely (implying an anticompetitive cross-market cost linkage), local liquidity deteriorates but global liquidity improves. The latter result is due to the intensified competitive pressure caused by the possibility to compete in only one venue. The opposite effect is found when the probability of having shocks of opposite sign is high.

Degryse et al. (2015) investigate the entry of Chi-X on the liquidity of Dutch stocks between 2007 and 2009. They find that fragmentation of transparent venues impairs local liquidity but improves global liquidity. These findings are consistent with our model predictions. Degryse et al. (2015) do not report any estimates of γ, but it is likely that it is above 1/3.[17] We would then be in the ultra-competitive case predicted by our model, in which the anticompetitive cross-market cost linkage is offset by the intensified competitive pressure caused by the possibility to compete in only one venue.

2.3.3 Interconnected liquidity

Proposition 2 shows that local expected spreads are indirectly influenced by orders sent to other venues due to the presence of strategic multi-venue market-makers. The latter make the liquidity of different venues interrelated in our model, as stated by the following Proposition:

Proposition 3 *Bid-ask spreads across venues co-vary jointly:*

$$Cov(s^D, s^S) = \zeta\big(\rho\sigma^2(I_u - I_d)\big)^2\left(\gamma \times g_{\phi_D}(\phi_S) + a_{\phi_D}\right) \tag{8}$$

where g_{ϕ_D} and a_{ϕ_D} are expressed in Appendix. Furthermore, $Cov(s^D, s^S)$ increases with γ.

Proposition 3 entails two main remarks. First, our model proposes a new explanation for inter-venue connectedness, namely the strategic quote placement by market-makers in response to non-constant marginal costs to supply liquidity across venues (termed as the cross-market

[17]Our own estimate of γ (see *d_POS* in Table 1) is 59% for a period using a sample of stocks (Dutch cross-listed stocks) and venues (Euronext) similar to Degryse et al. (2015).

cost linkage in our model). This explanation is distinct from those found in the literature which have focused on arbitrage strategies (Foucault et al., 2017; Tomio, 2017), duplicate strategies (van Kervel, 2015) or directional trading strategies (Chowdhry and Nanda, 1991; Baldauf and Mollner, 2017).[18]

Second, Proposition 3 shows that local bid-ask spreads co-vary more when γ increases. Kirilenko et al. (2017) suggest that the probability of having same-sign shocks is higher during period of crisis. Liquidity interconnectedness thus increases during period of market distress. This finding is also emphasized in our model by the quadratic dependency of the covariance to market-makers' risk aversion. During market distress, funding constraints or capital at risk constraints of market-makers are also more likely to increase, which translates in our model by an increase in market-maker's risk aversion, and a further rise in liquidity interconnectedness across venues. This result is consistent with the finding of Klein and Shiyun (2017) related to the increase in European liquidity betas during the 2008 financial crisis.

2.4 Testable implications

To establish the external validity of our modeling approach, we adopt a two-step empirical strategy. In the first step, we investigate whether cross-market inventory effects are present in the Euronext limit-order book environment. This step is meant to empirically validate our assumption that aggregate inventory is a driver of multi-venue market-making strategies.[19] In the second step, we proceed to test the main prediction of our model, derived from Proposition 1.[20]

[18] See Cespa and Foucault (2014) for interconnectedness across different assets.

[19] The literature has so far focused mostly on within-venue inventory effects in the context of dealer markets and the specialist-based model of the New York Stock Exchange (NYSE). See, among others, Hansch et al. (1998) and Reiss and Werner (1998) for London equity dealers, Bjønnes and Rime (2005) for foreign exchange dealers, or Panayidès (2007) for NYSE specialists.

[20] We acknowledge that our model with two strategic market makers competing in two transparent venues is very stylized. The model highlights however two new driving forces shaping price competition, which does not depend on any market structure. Even if the best prices would be different in case of competition between two limit order books, limit order traders would also face an "intense-competition" case versus a "low-competition" case.

2.4.1 Testing the validity of a cross-venue inventory model

Our model assumes that market-maker i' multi-venue quoting strategy is governed by her aggregate inventory, defined at time t as the cumulated net volume of transactions across all trading venues : $I_{i,t} = I_{i,0} + \sum_{\tau=0}^{\tau=t} Q_{D,\tau} + \sum_{\tau=0}^{\tau=t} Q_{S,\tau}$ where $I_{i,0}$ is the initial inventory. Our model implies that market-makers should react to a change in their aggregate inventory by adjusting quotes in *all* venues. In particular, after a trade, say in venue S, that increases the inventory exposure, a multi-venue intermediary should update quotes in venue S, but also in venue D to elicit inventory-reducing orders. We specifically focus on cross-venue inventory effects that, to the best of our knowledge, have never been investigated. Formulating our hypothesis in the context of the limit-order-book environment of Euronext, we test whether, for instance, after executing a sell order in the satellite venue that increases the total inventory exposure, a multi-venue market-maker is more likely to cancel an existing buy order in the dominant market, or modify it for a less aggressive price (negative revision), or post a new sell limit order in the dominant market or modify an existing sell order for a more aggressive price (positive revision). We thus posit the following hypothesis:

Hypothesis 1 *Multi-venue market-makers should update existing limit orders or submit new orders in one venue after a trade in another venue, in a direction that is associated with their inventory changes.*

We acknowledge that other trading strategies, such as cross-venue arbitrage, could lead to order placement patterns that resemble those due to inventory considerations. In case, say, the bid price in venue S jumps above the best ask in venue D, an arbitrageur might step in and sell one share in venue S, and buy one in venue D to reduce the existing price discrepancy. The buy and sell orders submissions from the arbitrageur are empirically similar to inventory-driven strategies. A way to distinguish these strategies is to take into account the aggressiveness of the initial transaction. In case there is an arbitrage opportunity, we expect arbitrageurs to post aggressive orders in a venue simultaneously/after an active transaction in another venue.[21] In contrast, after a passive transaction (existing limit orders passively hit), we expect more messages related to inventory management. We thus control for arbitrage opportunities and for the transaction aggressiveness in our empirical analysis.

2.4.2 Testing the main prediction of the model

Proposition 1 brings a novel prediction that relates price competitiveness to the signs of the shocks to absorb (same or opposite) and the divergence in intermediaries' inventories. In particular we expect tighter bid-ask spreads when competition gets more intense, i.e., when shocks in each venue have the same sign and the divergence in members' inventories increases (the ultra-competitive case). We thus formulate the following hypothesis:

Hypothesis 2 *Variations in spreads in one venue depend on the directions of order flows in both venues (identical or opposite), on the divergence in intermediaries' inventories, and on the interaction between the two.*

Note that bid-ask spreads vary more in the satellite venue than in the dominant venue, due to a larger impact of the cross-market cost linkage and due to larger variation in competition intensity. Recall that, despite a shock of a smaller magnitude, the best ask price in the satellite venue may be higher than the one in the dominant venue (low-competition region to the left of the point p on Figure 3). In contrast, when divergence is high (ultra-competitive region to the right of the vertical line Q_D), competition heats up and the best ask price in the satellite venue is smaller than in the dominant venue (reflecting the smaller quantity to absorb).

This prediction is novel and interesting because it allows us to depart from a competing adverse-selection hypothesis and from a competing pure risk-sharing hypothesis. First, in case a (fast) informed trader with simultaneous access to all venues would split his orders across venues, the adverse selection component of multi-venue market-makers should increase. Market-makers

[21] We call a transaction "active" when intermediaries trade through a liquidity demanding order like a market or marketable order.

should reduce their liquidity supply in all venues (van Kervel, 2015). Quoted bid-ask spreads should thus increase in all venues if order flows across venues have same direction, like our cross-market cost linkage. Our model however predicts that adding an interaction term between the direction of order flows and a measure of divergence in inventories should have a negative impact on spreads, unlike the adverse-selection hypothesis. Secondly, in case market makers would behave competitively, or in case they would not face non constant marginal costs across venues, the interaction term would not impact spreads variations.[22]

3 Empirical Analysis

3.1 Forming the sample

Our analysis uses a proprietary dataset from Euronext on multi-listed stocks. Euronext was created in 2000 as a result of the merger of three European exchanges, namely Amsterdam, Brussels and Paris. The Lisbon exchange joined in 2002.[23] Before the introduction of the Universal Trading Platform (UTP) in 2009, each of the four exchanges maintained their domestic market. As a result, firms could be multi-listed on several Euronext exchanges; for example, Suez was traded in Paris and Brussels.

Our sample consists of all multi-traded stocks within Euronext, spanning four months (79 trading days) from January 1, 2007 to April 30, 2007.[24] The data on orders and quotes are provided by Euronext. Euronext files also provide us with the identification of the member participating in each quote or transaction, and whether the member is acting as an agent or as a principal (that is, either as a proprietary trader or an exchange-regulated market maker). The data assigns a unique identifier to each member, enabling us to trace members' inventory changes and quoting behavior across time, across stocks, and across exchanges. During the

[22] In the Online Appendix, we suppose that market-makers behave competitively. Appendix A shows that the ultra-competitive case is not obtained for same-sign shocks and a high divergence among market-makers' inventory positions.

[23] Euronext has expanded to five exchanges with the acquisition of the Dublin exchange in 2017.

[24] Four trading days are dropped in January due to missing data about best limits.

sample period, Euronext exchanges followed the same market model (same trading hours, same trading fees, and same trading rules), and the payment of membership fees granted access to all Euronext markets. Note also that, during this period (pre-MiFID environment), trading was concentrated in Euronext.[25] For all these reasons, Euronext is an excellent environment to test the predictions of our model. Other stock-level information comes from Compustat Global.

We keep firms that trade in euros using a continuous trading session in at least two exchanges on which they are traded. To avoid introducing threshold effects, we follow a conservative approach and keep all members who trade at least once in each of the two exchanges on which the stock is traded. Overall, we follow 46 multi-venue members. Because these members do not necessarily follow the same stocks, our sample finally consists of 178 pairs (stock, member), among which 20% involve an exchange-regulated market-maker (registered as such in at least one market on which the stock is traded), called thereafter Designated Market-Maker (DMM) (see Panel C of Table 1).[26]

The final sample contains 20 firms with at least one multi-venue member, trading continuously in two Euronext exchanges. Among them, 11 are traded on Euronext Amsterdam, 12 are traded on Euronext Brussels and 17 on Euronext Paris. To determine which is the dominant market (market D in the model) and which is the satellite market (market S in the model), we use the primary market as the (exogenous) dominant platform.

3.1.1 Measuring liquidity

We measure the spread in the market m as the equally-weighted average bid-ask spread for stock j, during a twenty-minutes interval t.[27] We focus on the relative bid-ask spread $RBAS\_m$,

[25] Some French stocks were traded on the London Stock Exchange or the Deutsche Böerse, while some Dutch stocks were traded on Xetra. Gresse (2017) finds a market share of 96.45% for CAC40 stocks and even 99.99% for other SBF120 stocks in October 2007. Degryse et al. (2015) also find that Dutch stocks are overwhelmingly traded on Euronext.

[26] Our paper does not compare the liquidity provision of exchange-regulated market-makers versus endogenous market-makers, as Anand and Venkataraman (2016) do using Canadian data. We however keep trace of difference in trading behaviors.

[27] We compute both equally-weighted and time-weighted averages of the quoted spreads. As the results for the two weighting schemes are virtually identical, we restrict the presentation to the equally-weighted spread measures.

and the variation of the relative spread between two consecutive intervals, $\Delta RBAS\_m$, where $m = DOM, SAT$.

3.1.2 Measuring aggregate inventory

In our dataset, the initial inventory position (I_0) of members is not observable. Moreover, members differ in the amount of capital at risk they commit to their trading activities and/or in their tolerance for risk, which makes inventories not comparable to each other. We thus follow Hansch et al. (1998) methodology by building standardized inventory positions to deal with these unobservable characteristics. Let $IP^s_{i,t}$ denote the inventory position of multi-venue member i in stock s at time t. We use the record of all trades executed by i in all venues, plus the direction of these trades to obtain her aggregate or net inventory position.We thus construct a time series for each member's inventory position in each stock across all Euronext venues from the start to the end of our sample period. Since at the time more than 95% of the volumes were traded on Euronext, our inventory variable is a good proxy for intermediaries' aggregate inventories. We compute the mean ($\overline{IP}^s_i$) and the standard deviation (σ_i) for each of these inventory series. The standardized inventory is defined as

$$I^s_{i,t} = \frac{IP^s_{i,t} - \overline{IP}^s_i}{\sigma^s_i}.$$

We then build a measure of divergence in inventories. Let $I^s_{M,t}$ denote the median inventory at time t in stock s, and let $ID_{i,t} = |I^s_{i,t} - I^s_{M,t}|$ denote the member i's inventory position relative to the median inventory. The larger ID_i, the more divergent the inventory of member i relative to the median is, and the more competitive her quotes are, in order to reduce her inventory exposure (Hansch et al., 1998). We take the mean of inventory divergence across intermediaries at time t in each stock s, $\overline{RI}^s_t$, to get a proxy of divergence in intermediaries' inventories ($I_1 - I_2$ in our model).

3.1.3 Determining the direction of order flows across venues

The model's predictions depend on whether liquidity demands sent across venues have the same or the opposite direction. We proxy liquidity demand by the net order flow in market m (i.e., trade imbalance) in stock s during a twenty-minutes interval, $TrIMB\_m$, as the number of buyer-initiated trades minus the number of seller-initiated trades.[28] Trade imbalance is positive is there are at least as many buy initiated trades as sell initiated trades, and negative if there are strictly more sell initiated trades than buy initiated trades. The dummy variable d_POS is defined when there are trades in both markets; it takes the value of one if order flows have the same direction across venues ($TrIMB\_DOM \times TrIMB\_SAT > 0$) on a given twenty-minutes interval, and zero if order flows have opposite signs across venues. Note that we exclude the first and last five minutes of trading in order to avoid contamination by specific trading behaviors during the open or close of the markets.[29]

3.1.4 Control variables

In our regression specifications, we control for the existence of arbitrage opportunities. This is necessary because, by buying the asset in one venue and reselling it in the other venue, arbitrageurs behave as inventory-driven market-makers. We only consider realized arbitrage opportunity. The dummy d_AO takes the value of one if the best bid in one venue exceeds the best ask in the other venue, i.e., $max(Bid\_SAT, Bid\_DOM) > min(Ask\_SAT, Ask\_DOM)$ and if two opposite trades occur at these prices. We also expect arbitrageurs to use more often active transactions (marketable orders) than passive transactions (non-aggressive limit orders) to take fast arbitrage opportunities. We thus use the dummy d_AT which takes the value of one if the origin of transaction executed by the member is a market/marketable order, and zero if it is a limit order hit. In some regressions, we also control for the pending time to the next market close ($TimeClos$), the (log) transaction size in number of shares ($TrSize$), and the number of

[28] Note that our data specify the sign of trades.

[29] On February 19, 2007, the closing fixing moved from 5:25 pm to 5:30 pm. We therefore drop all observations before 9:05 am and after 5:20 pm.

trades $NbTr$.

3.2 Summary statistics

Table 1 presents summary statistics for our sample. Panel A presents statistics across stocks. The average (median) firm has a stock price of 53.3 (50.09) Euros, a market cap of 30.6 (20.4) billion Euros, and 9 (5) multi-venue members trading on the stock. There is an average number of 3 realized arbitrage opportunities per day, and 59% of order flows across venues have the same direction. Panel B presents statistics computed within each market. Relative (quoted) spreads of the satellite market are five to ten times larger than those of the dominant market, depending if one takes means or medians. The daily number of trades is much smaller (twenty five times less in average) in the satellite market, reflecting lack of trade activity, and transaction size is also much smaller. Surprisingly, the daily number of best limit updates is only three times less in average in the satellite venue. This suggests that the satellite market is not a very active trading place, but it is closely monitored. T-tests of the difference in means between the two markets (not shown) confirm the statistical significance of these differences. Panel C presents statistics computed for each multi-venue member. There is considerable heterogeneity in terms of member trading activity, resulting from our conservative selection. The average multi-venue member makes 70 trades per day in the dominant market and 9 trades in the satellite market, but the median member only does 8 and 1 respectively. Panel C also shows the mean reversion parameter in members' aggregate inventory, obtained by estimating the following regression model of inventory time series for each pair (stock, member),

$$\Delta I_{it} = \alpha + \beta I_{it-1} + \varepsilon_t,$$

where ΔI_{it} is the change in aggregate inventory from the previous trade. Mean reversion predicts that $\beta < 0$ (if $\beta = 0$, it has a unit root and it is non-stationary). Across the 178 pairs, Panel C shows that the average mean-reversion parameter (β) is -0.073, which means that multi-venue members reduce, in average, inventory by 7.3% during the next trade.

3.3 Multivariate analysis

3.3.1 Inventory management across venues

The first step of our empirical analysis is to validate that aggregate inventory matters for multi-venue members. Panel C already shows that aggregate inventories of some members are mean-reverting, which is consistent with the model. We now investigate whether a multi-venue member sends inventory-driven messages in one venue in response to a transaction in another venue (that is, a transaction that causes a change in her aggregate inventory). We focus on messages routed to the dominant market after a transaction in the satellite market, because effects in the more liquid market should be more easily detected. For example, after a buy in the satellite market, a multi-venue member should cancel or negatively revise existing buy orders – or submit new sell orders or positively revise sell orders in the dominant market. The opposite should occur after a sell. We implement the following Logit regression:

$$\begin{aligned} Pr(d\_i_\tau) &= \alpha + \beta_1 d\_DMM + \beta_2 |I_{i,\tau-1}| + \beta_3 d\_DMM \times |I_{i,\tau-1}| \\ &\quad + \beta_4 d\_AO_\tau + \beta_5 log(TrSize_\tau) + \beta_6 TimeClos_\tau + \varepsilon_\tau, \end{aligned} \tag{9}$$

where d_i is the dummy variable that takes 1 if member i sends a message in the dominant market in direction of inventory following a trade at time τ in the satellite market.[30] The explanatory variables are the lagged absolute inventory position of member i, the dummy variable for designated market-makers, and the interaction between both. We control for the existence of an arbitrage opportunity at the time of the trade, the size of the trade, and the pending time to the close. Our specification also includes firm fixed-effects to control for time-invariant firm heterogeneity. We run the regression both after an active and a passive transaction.

The results of the Logit analysis are presented in Table 2. Panel A reports the results

[30] Messages are tracked through their first 10 seconds after a trade.

for order submissions after a passive transaction, while Panel B reports the results for order submissions after an active transaction. First, in both cases, the likelihood that multi-venue members use "inventory-driven" strategies is larger when they are dedicated market-makers. Second, these trading strategies seem different according to whether the change in aggregate inventory has been caused by a passive transaction or an active transaction, consistently with the discussion of Hypothesis 1. The probability to post cross-venue inventory-driven messages is negatively related to the existence of an arbitrage opportunity when the transaction is passive, while it is significantly positively related when it is active.

In particular, Panel A shows that, when the transaction is passive, dedicated market-makers are more likely to use cross-venue inventory-driven messages, even more likely when their aggregate inventory is large. This finding validates the assumption of the model that multi-venue members manage inventory risk across multiple venues. When the transaction is active, Panel B shows that the coefficients of the dummy Arbitrage Opportunity and the dummy for designated market-maker are positive and significant. This suggests that multi-venue designated market-makers take arbitrage opportunities by posting aggressive orders in the two venues. This is in line with the role that Euronext assigns to designated market-makers in cross-listed stocks. Note that, in this case, the aggregate inventory of dedicated market-makers has no significant impact, supporting the notion that the observed sequence of messages is driven by an arbitrage trading strategy.

In summary, these results are consistent with Hypothesis 1 of multi-venue members using cross-venue strategies to manage inventory aggregated over all venues.

3.3.2 Spreads

To test the main prediction of our model (Hypothesis 2), we estimate the relation between the variation in twenty-minute bid-ask spreads in the satellite market and the intensity of price competition among multi-venue members which is related to the divergence in their inventories

($\overline{RI}^s$), to the direction of order flows across venues (i.e., whether the dummy d_POS is equal to one), and to the interaction between the two. We run the following panel regression model:

$$\Delta RBAS\_SAT_t^s = \alpha + \beta_1 d\_POS_t^s + \beta_2 \overline{RI}_{t-1}^s + \beta_3 d\_POS_t \times \overline{RI}_{t-1}^s + \beta_4 NbTr\_SAT_t^s + \varepsilon_t^s. \quad (10)$$

Proposition 1 predicts that the sign of the order flows routed across venues impacts the spreads. More specifically, we expect the following sign: $\beta_1 > 0$ due to the anticompetitive effect of the cross-market cost linkage when shocks hitting venues have the same sign. We also expect that in case of both a large divergence in inventories and same-sign shocks, at least one member competes more intensely to execute all orders across venues, implying $\beta_3 < 0$. This interaction term allows us to distinguish our predictions from those of an adverse selection model, since the latter would predict $\beta_3 \geq 0$. Finally, the number of trades in the satellite market, $NbTr\_SAT$, controls for the activity in the satellite market.

All specifications include day dummies and use clustered standard errors by stock to accommodate the possibility that relative spreads are strongly correlated within firms.

Table 3 presents estimation results. We report two specifications: the first with time fixed effects (Column 1) and the second with day and firm fixed-effects. The main conclusions from the analysis are as follows. First, spreads in the satellite market vary with the direction of order flows across venues (coeff. 0.108, t-stat. 2.14 in column 1). This result is consistent with the cross-market cost linkage Second, the variable of interest which is the interaction term between same-sign order flows and divergence in inventories has a negative and statistical significant impact on spreads changes (coeff. -0.12, t-stat. -2.00). Estimates in columns (1) and (2) imply that a one-standard deviation shock in the divergence in inventories ($\overline{RI}$) is associated with a negative change of 0.9 basis point in relative spreads.[31] Spreads in the satellite market are thus significantly lower when there exists members holding large aggregate inventory and when order flows across venues have the same sign, supporting Hypothesis 2. This result is consistent with

[31] The average change in relative spreads is 0.46 basis point. The standard deviation of the variable $\overline{RI}$ is 0.2678.

the case of intense competition among intermediaries illustrated by Regions B and C of Figure 4, and uniquely predicted by our model. Results for other control variables are not statistically significant. Overall, the results in Table 3 corroborate the main implication of the model.

4 Conclusion

We develop a two-venue duopoly model in which prices posted in each venue by fast market-makers are guided by their net inventory position aggregated across all venues. This implies that executing a trade in one venue simultaneously changes marginal costs to provide immediacy in all other venues. This cross-market cost linkage is a new and additional channel altering competition in a fragmented market. Moreover, strategic market-makers are not forced to undercut in each venue, but strategically choose to compete for the shock they would like to absorb. We show that the cross-market cost linkage and the possibility to undercut in only one venue may increase competition and enhance liquidity.

In our model, local bid-ask spreads depend: (i) on whether demand shocks hitting venues have same sign; (ii) on whether at least one market-maker holds an extreme inventory position; and (iii) on the interaction between the two. We exploit the co-existence of multiple identical order books for the same security within Euronext (before 2009) to test our model. First, we uncover new evidence of cross-venue inventory effects. Second, our panel regression analysis reveals that local bid-ask spreads vary in a way which is uniquely predicted by our competition model.

Our results suggest that the cross-market inventory cost linkage is an alternative mechanism to the information channel that explains common factors in liquidity. Effects could be emphasized if we now consider a market participant trading a portfolio of assets with correlated returns. Market-makers quotes' placement across venues should take into account her aggregate inventories in all assets in portfolio and how they fluctuate together. The impact of multi-venue multi-asset market-making raises challenging questions related to liquidity spillovers across assets and across venues. While this is an issue outside the scope of this paper, we believe it is an interesting topic for future research.

Figure 1: One day of two-venue quotes placement and aggregate inventory of a Euronext multi-venue intermediary trading Suez

Figure 1 plots the aggregate inventory of a Euronext intermediary trading Suez and the prices that she posts on Euronext Paris and Euronext Brussels, compared to the midpoint during that trading day, January 19, 2007. The intermediary is a formally registered market-maker in Suez. The top graph plots three series of prices. The pink dash-dotted line plots the midpoint computed as the average between the consolidated best ask and best bid, i.e., the lowest ask (resp. the highest bid) across the dominant and the satellite market. The hollow circles depict the prices that the market-maker posts in the satellite market while the dark-blue triangles depict her quotes in the dominant market. Euronext Paris and Euronext Brussels are limit order books: the figure only depicts the liquidity supply activity of the market-maker (limit order placement). The bottom graph plots the aggregate euro inventory of the market-maker for the day, which is computed using the record of all signed market-makers' trades multiplied by the price of transaction across all trading venues.

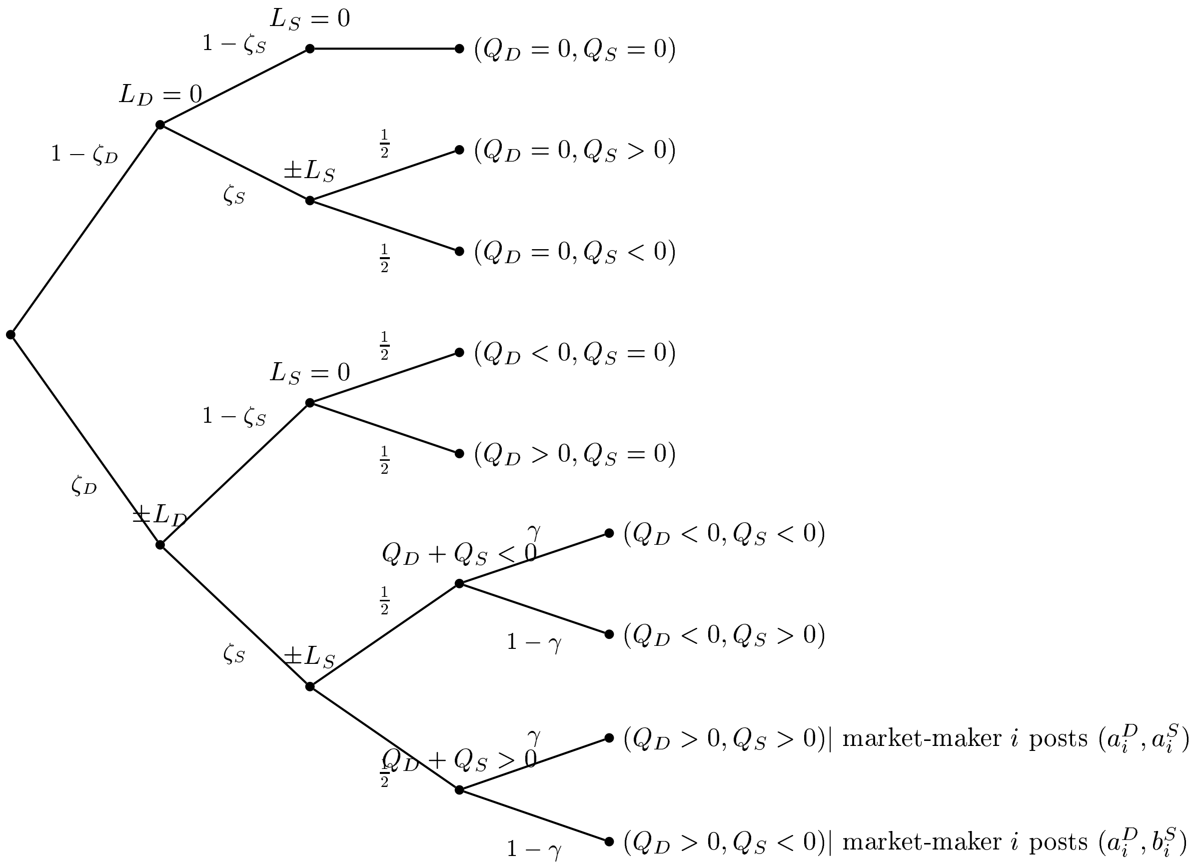
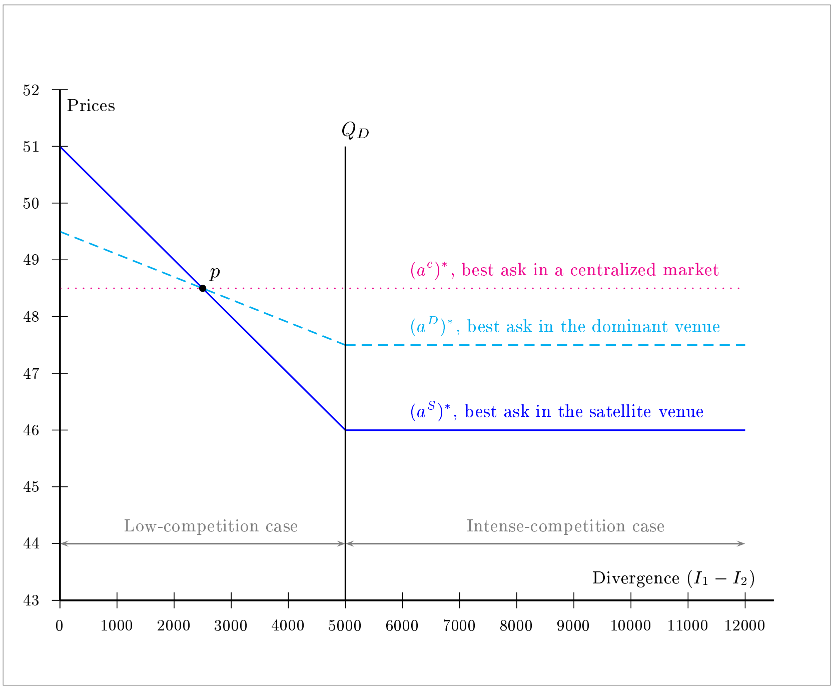
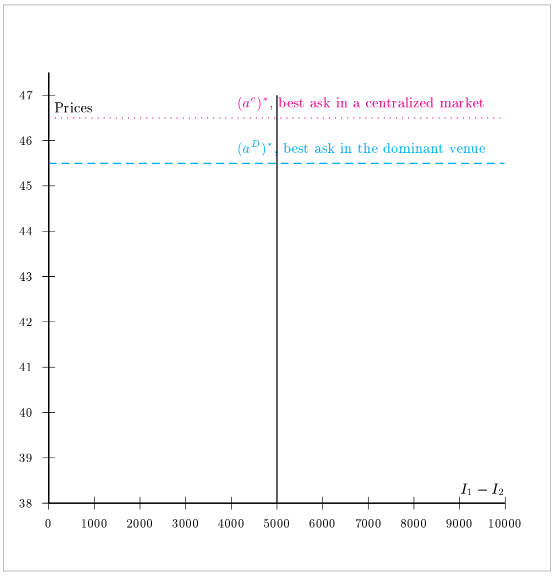


Figure 2: Tree of the quoting game across trading venues

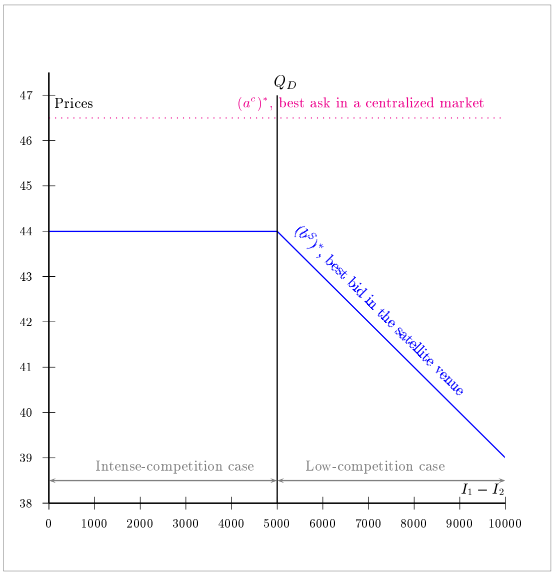
Figure 2 represents the tree of the trading game. At date 1 (not represented on the Figure), market-maker i is endowed with an inventory position denoted I_i. At date 2, venue m is hit by a liquidity shock, denoted L_m, with probability ζ_m. L_m generates a liquidity demand Q_m, which is positive (resp. negative) with probability $\frac{1}{2}$ (resp. $\frac{1}{2}$). The probability that shocks simultaneously hit both venues is denoted ζ ($= \zeta_D \times \zeta_S$). The probability that shocks have the same sign is denoted γ. The paper analyzes price formation across venues when the global order flow is net-buying, i.e., $Q_D + Q_S > 0$. Symmetric results are obtained for a net-selling global order flow. At date 3, market-maker i posts simultaneously a price in venue D and a price in venue S. We denote a_i^m (resp. b_i^m) the ask price (resp. bid price) that i posts in venue m if $Q_m > 0$ (resp. $Q_m < 0$), $m = D, S$.



(Panel A)



(a)



(b)

(Panel B)

Figure 3: Illustration of Proposition 1

Figure 3 illustrates Proposition 1. Panel A shows equilibrium selling prices in a fragmented market when buy shocks hit simultaneously venues D and S. Panel B depicts best prices when a buy shock hits venue D (Panel B (a)) and a sell shock hits venue S (Panel B (b)). The dotted magenta line depicts the best ask price in a centralized market, the cyan dashed line plots the best selling price in venue D, and the plain blue line plots the best ask (Panel A) or best bid (Panel B) price in venue S depending on the sign of the shock hitting S. In case of two positive shocks the intersection point of the 3 equilibrium prices ($(a^c)^*$, $(a^D)^*$ and $(a^S)^*$), termed p, is also represented in Panel A. p is such that $I_1 - I_2 = \frac{Q}{2}$. The vertical line Q_D separates the region in which there is a low divergence in market-makers' inventories ($I_1 - I_2 \leq Q_D$) from the region in which there is a high divergence in inventories ($I_1 - I_2 > Q_D$). Parameters are $Q_D = 5,000$, $|Q_S| = 2,000$, $I_u = 15,000$, $I_d = 0$, $\mu = 50$, $\sigma^2 = 0.001$, $\rho = 1$, $I_2 = 5,000$, I_1 is varying.

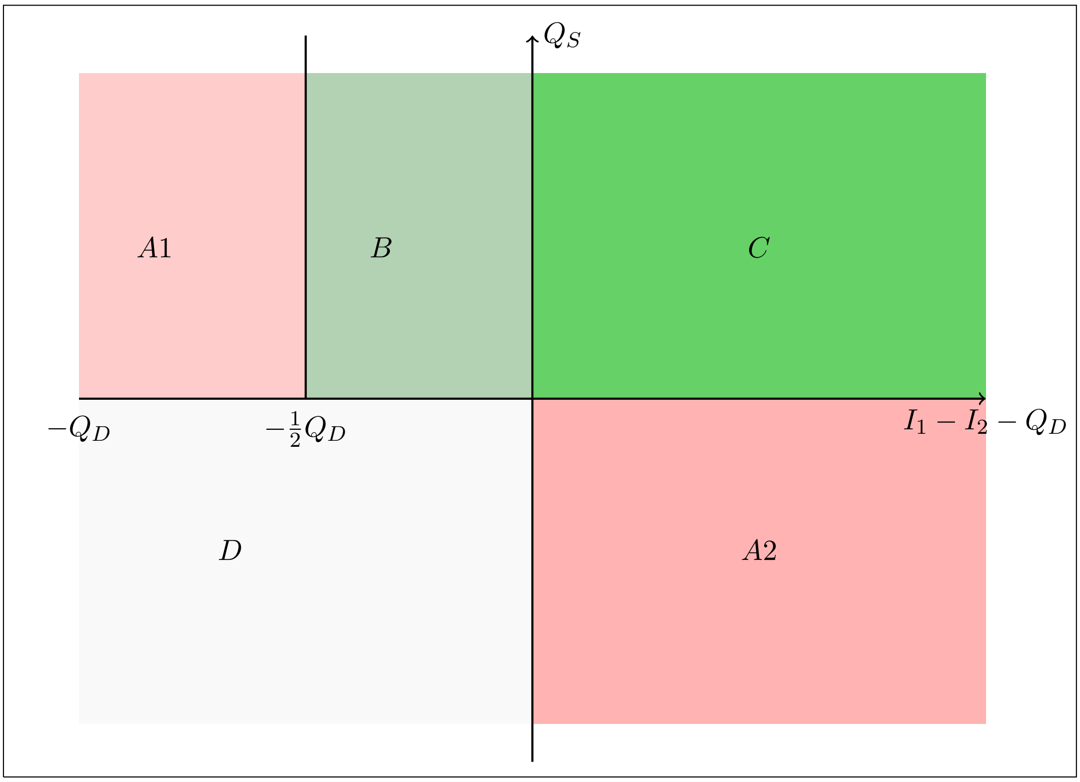


Figure 4: Is market fragmentation good for transactions costs?

Figure 4 partitions the set of parameters into 5 different regions. The y-axis represents the shock hitting S, Q_S, which might be positive or negative and the x-axis represents $I_1 - I_2 - Q_D$ varying from $-Q_D$ to $I_u - I_d - Q_D$. The product $(I_1 - I_2 - Q_D) \times Q_S$ determines whether market-maker 1 is capacity-constrained (see Lemma 1). The vertical line $-\frac{1}{2}Q_D$ is such that prices are identical across all venues or market structures (see comments on p in Figure 3 Panel A). The red regions $A1$ and $A2$ are such that ex post transaction costs are higher in a fragmented market ($TTrC - TTrC^c > 0$). In the other 3 regions (B, C, and D) ex post transaction costs are equal or strictly smaller than those in a centralized market. The neutral region D is such that fragmentation is innocuous: $TTrC - TTrC^c = 0$. Region B in light green is such that transaction costs are lower in a fragmented market ($TTrC - TTrC^c \leq 0$). Region C in dark green is such that transaction costs are strictly lower in a fragmented market ($TTrC - TTrC^c < 0$) (region of the "ultra-competitive case").

Table 1

Summary Statistics

This table reports summary statistics for the data used in this study. The sample consists of 20 multi-listed, continuously-traded stocks on Euronext exchanges, from January 1, 2007 through April 30, 2007 (79 trading days). The quotes and trades data comes from Euronext, and other stock-level information comes from Compustat Global.

Panel A reports the daily mean across the 20 stocks for the variables used in this study. Market capitalization is price times shares outstanding, in millions of Euros. Number of Trades is the number of transactions per day across the total number of trading venues. Number of Messages is the daily total number of orders (submissions, revisions, cancellations) across the total number of trading venues. Trade Size is the daily average size of transactions across trading venues. Number of Realized Arbitrage Opportunities is the daily number of times the best bid in the dominant (resp. satellite) market is greater than the best ask in the satellite (resp. dominant) market and buy and sell trades by the same intermediary are observed during the window of the arbitrage opportunity. Number of multi-venue intermediaries is the total number of market-makers as defined in Section 3.1. Average Inventory Divergence (RI_m) is the average divergence in market-makers' inventories, where inventories are measured each 20 minutes interval. d_POS is a dummy variable that takes the value of one if order flows across venues have the same direction.

Panel B reports summary statistics by market type. It contains news variables. Bid-Ask Spread is the equally-weighted average difference between the best bid and the best ask during the day. Relative Spread is equal to the equally-weighted average of ratio between the spread and the midpoint. Number of Best Limits Updates is the total number of times there is a change in the best limits. Percentage of Active Trades is the ratio of the number of transactions caused by a market or a marketable order over the total number of transactions in the trading venue. Percentage of Passive Trades is the ratio of the number of limit order hit over the total number of transactions in the trading venue. Percentage of Cancellations (resp. New Submissions) is the ratio of the number of cancellations (resp. new submissions) over the total number of messages in the trading venue. Percentage of Revisions is the ratio of the number of revised orders (messages other than new submissions and cancellations) over the total number of messages in the trading venue.

Panel C reports summary statistics by multi-venue intermediaries. d_DMM is the dummy that take one if the multi-venue intermediary is an exchange-regulated market-maker, also called Dedicated Market-Maker (DMM) in the stock. Number of Trades in D is the average daily number of transactions executed in the dominant venue. Number of Trades in S is the average daily number of transactions executed in the satellite venue. Percentage of Passive Transactions in S is the ratio of the number of limit order posted by the intermediary i which are hit in the satellite market over the total number of transactions. Percentage of Messages in Direction of Inventory is the ratio of the number of messages submitted within 10 seconds in the dominant market after a transaction in the satellite market which are in direction of inventory management over the total number of messages submitted within 10 seconds in the dominant market after a transaction in the satellite market. Delay to submit a message in direction of inventory is the number of second between a transaction in S and an inventory-driven message in D.

Table 1

Summary statistics (cont.)

Panel A. Summary statistics by stock

	N	Mean	Std. Dev.	Q1	Median	Q3
Market Capitalization (in billion)	1197	30589	33500	2396	20438	50089
Price	1577	53.30	36.40	25	50	70
Number of Trades	1577	2645	3142	73	1635	4213
Number of Messages	1577	9850	9924	1524	7079	15355
Trade Size	1553	491	576	33	304	1617
Number of Arbitrage Opportunities	1577	3	9	0	0	3
Number of multi-venue intermediaries	1577	9	9	3	5	10
Average inventory divergence, RI_m	1577	.62	.36	.38	.59	.82
d_POS	1224	.59	.29	.45	.60	.76

Panel B. Summary statistics by venue

B.1 Dominant venue

	N	Mean	Std. Dev.	Q1	Median	Q3
Bid-Ask Spread	1577	.11	.13	.022	.06	.16
Relative Bid-Ask Spread	1577	.28	.37	.07	.12	.27
Number of Best limits Updates	1577	6059	5095	776	4847	9655
Number of Trades	1577	2577	3108	73	1449	4055
Percentage of Active Trades	1577	45	26	28	39	54
Percentage of Passive Trades	1577	55	26	45	60	72
Percentage of Cancelations	1407	12	13	0	9	21
Percentage of Revisions	1407	33	36	4	17	60
Percentage of New Submissions	1407	22	17	5	25	34
Transaction Size	1577	620	684	192	360	779

B.2 Satellite venue

	N	Mean	Std. Dev.	Q1	Median	Q3
Bid-Ask Spread	1564	1.24	2.38	.066	.33	1.55
Relative Bid-Ask Spread	1564	1.87	3.28	.24	1,00	1.98
Number of Best limits Updates	1551	2614	3797	81	794	4040
Number of Trades	1109	95	385	0	3	20
Percentage of Active Trades	1109	31	28	0	30	45
Percentage of Passive Trades	1109	69	28	55	70	100
Percentage of Cancelations	1395	8	11	0	4	10
Percentage of Revisions	1395	79	26	70	90	98
Percentage of New Submissions	1395	8	12	0	4	11
Transaction Size	1109	348	369	100	250	485

Panel C. Summary statistics by multi-venue intermediary

	N	Mean	Std. Dev.	Q1	Median	Q3
Dummy for Dedicated Market-Maker	178	0,19	0,39	0,00	0,00	1,00
Average Mean Reversion of Inventory	178	-0,073	0,150	-0,314	-0,013	0,001
Number of Trades in D	178	70	131	0	8	377
Number of Trades in S	178	9	28	0	1	69
Percentage of Messages in Direction of Inventory	110	66	30	0	66	100
Percentage of Passive Transactions in S	178	53	30	0	52	98
Delay to submit a message in Direction of Inv.	110	3	2	0	3	8

Table 2

Likelihood of Expected Inventory-driven Message

following a Transaction in the Satellite Market

This table presents estimates of the relation between the likelihood of an inventory-driven message posted by the intermediary i in the dominant market after a trade in the satellite market. The left-hand side variable is Indicator of Expected Message, a dummy variable that takes the value 1 if the message has the expected value. Left-hand side variables are described in caption of Table 1. $DMM\times$Standardized Inventory is an interaction term equal to the product of DMM and Standardized Inventory. Panel A shows regression specifications in the subsample of passive transactions. Panel B shows regression specifications in the subsample of active transactions. All specifications include firm fixed effects and t-statistics are calculated using standard errors clustered by liquidity supplier. The symbols ***, **, * denote significance levels of 1%, 5% and 10%, respectively for the two-tailed hypothesis test that the coefficient equals zero.

Panel A. Passive Transactions				
Dependent variable:	Indicator of Expected Message			
	(1)		(2)	
Log Trade Size	0.032		0.032	
	(1.05)		(1.05)	
Standardized Inventory	0.018		-0.02	
	(0.56)		(-0.55)	
DMM	1.522	***	1.377	***
	(3.70)		(3.42)	
Arbitrage Opportunity	-0.310	***	-0.309	***
	(-3.31)		(-3.33)	
Time to close	0.025		0.025	
	(1.38)		(1.36)	
DMM × Standardized Inventory			0.187	**
			(2.33)	
Intercept	0.217		0.243	
	(0.66)		(0.74)	
Firm FEs	Yes		Yes	
N	18,022		18,022	
Pseudo R^2	0.06		0.06	

Panel B. Active Transactions				
Dependent variable:	Indicator of Expected Message			
	(1)		(2)	
Log Trade Size	-0.015		-0.014	
	(-0.45)		(-0.45)	
Standardized Inventory	-0.005		0.043	
	(-0.08)		(0.59)	
DMM	0.646	**	0.733	***
	(2.44)		(3.76)	
Arbitrage Opportunity	0.597	***	0.603	***
	(4.46)		(4.58)	
Time to close	0.013		0.014	
	(0.80)		(0.81)	
DMM × Standardized Inventory			-0.125	
			(-0.67)	
Intercept	1.402	**	1.348	**
	(2.30)		(2.10)	
Firm FEs	Yes		Yes	
N	9,100		9,100	
Pseudo R^2	0.06		0.06	

Table 3

Determinants of Relative Spreads in the Satellite Market

This table presents estimates of the relation between changes in relative bid-ask spreads in the satellite market and the divergence in intermediaries' inventories and the direction of order flows across venues. The left-hand side variable is the Change in Relative Spread of the Satellite market in the 20-minutes interval. The right-hand-side variables are defined in caption of Table 1. $d_POS \times$Lag AbsoluteRI is an interaction term equal to the product of the dummy of same-sign shocks (d_POS) and Lag AbsoluteRI. t-statistics are calculated using standard errors clustered by firm. The symbols ***, **, * denote significance levels of 1%, 5% and 10%, respectively for the two-tailed hypothesis test that the coefficient equals zero.

Dependent variable:	Change in Relative Spread of Market S			
	(1)		(2)	
d_POS	0.108	**	0.105	**
	(2.14)		(2.13)	
Lag Absolute RI	0.087		0.076	
	(1.14)		(1.34)	
d_POS × Lag Absolute RI	-0.12	**	-0.119	**
	(-2.00)		(-2.01)	
Number of Trades in Market S	-0.050		0.004	
	(-1.30)		(0.12)	
Intercept	-0.078		-0.065	
	(-0.93)		(-1.03)	
Time FEs	Yes		Yes	
Firm FEs	No		Yes	
N	11,172		11,172	
Adjusted R^2	0.01		0.03	

A Appendix

A.1 Intermediaries' trading profits

Market-maker i's trading profit is given by:

$$V_i\left(p_1^D, p_2^D, p_1^S, p_2^S\right) = \begin{cases} \underbrace{p_i^D Q_D + p_i^S Q_S - TC_i(Q_D + Q_S)}_{\equiv v_i(Q_D+Q_S)} & \text{if } p_i^D Q_D < p_{-i}^D Q_D \text{ and } p_i^S Q_S < p_{-i}^S Q_S, \\ \underbrace{p_i^D Q_D - TC_i(Q_D)}_{\equiv v_i(Q_D)} & \text{if } p_i^D Q_D < p_{-i}^D Q_D \text{ and } p_i^S Q_S > p_{-i}^S Q_S, \\ \underbrace{p_i^S Q_S - TC_i(Q_S)}_{\equiv v_i(Q_S)} & \text{if } p_i^D Q_D > p_{-i}^D Q_D \text{ and } p_i^S Q_S < p_{-i}^S Q_S, \\ 0 & \text{if } p_i^D Q_D > p_{-i}^D Q_D \text{ and } p_i^S Q_S > p_{-i}^S Q_S. \end{cases}$$

where $TC_i(Q)(= r_i(Q) \times Q)$ denotes the inventory costs to absorb the shock Q for market-maker i ($Q = Q_S, Q_D$ or $Q_D + Q_S$), p_i^D denotes the price set by market-maker i in venue D, and p_i^S denotes the price posted by i in venue S, $i = 1, 2$. The price p_i^m is an ask price if $Q_m > 0$ and a bid price if $Q_m < 0$, $m = D, S$.[32]

A.2 Proof of Lemma 1

We consider two cases separately.

Case 1 ("Virtual consolidation"). We first look for the necessary conditions to be simultaneously filled to guarantee the existence of an equilibrium in which a single market-maker simultaneously absorbs the shock in the dominant venue and the shock in the satellite venue.

Market-maker $i \in \{1, 2\}$ executes the global order flow in equilibrium if and only if she simultaneously posts the best price in the dominant venue and the satellite venue. The lowest ask price a_i^D prevailing in venue D, and the lowest ask price p_i^S (resp. highest bid price) prevailing in venue S when $Q_S > 0$ (resp. when $Q_S < 0$) are such that:

- i: trading $Q_D + Q_S$ is profitable for market-maker i (i.e., $v_i(Q_D + Q_S) \geq 0$), and (i') not for market-maker $-i$ (i.e., $v_{-i}(Q_D + Q_S) < 0$);
- ii: trading $Q_D + Q_S$ is more profitable for market-maker i than trading only Q_D (i.e., $v_i(Q_D + Q_S) \geq v_i(Q_D)$), or (ii') only Q_S (i.e., $v_i(Q_D + Q_S) \geq v_i(Q_S)$);
- iii: undercutting market-maker i is not profitable for market-maker $-i$ neither in venue D (i.e., $v_{-i}(Q_D) < 0$), nor (iii') in venue S (i.e., $v_{-i}(Q_S) < 0$).

Using the expression of market-makers' trading profits V, this set of conditions rewrites as follows:

$$\begin{aligned} \text{i} &: a_i^D Q_D + p_i^S Q_S \geq TC_i(Q_D + Q_S), \\ \text{i'} &: a_{-i}^D Q_D + p_{-i}^S Q_S < TC_{-i}(Q_D + Q_S); \\ \text{ii} &: a_i^D Q_D + p_i^S Q_S - TC_i(Q_D + Q_S) \geq a_i^D Q_D - TC_i(Q_D), \\ \text{ii'} &: a_i^D Q_D + p_i^S Q_S - TC_i(Q_D + Q_S) \geq a_i^S Q_S - TC_i(Q_S); \\ \text{iii} &: a_i^D Q_D < TC_{-i}(Q_D), \\ \text{iii'} &: p_i^S Q_S < TC_{-i}(Q_S). \end{aligned}$$

[32] As in Biais (1993), the utility function of intermediaries given in Eq. (1) is linearized, under the assumption $Q_D < I_u - I_d$. Note that, in our transparent setting, the criticism on the linear approximation used by Biais (1993) for opaque markets raised by de Frutos and Manzano (2002) does not apply. The assumption $Q_D < I_u - I_d$ also guarantees that market-maker i has a probability to post the best price in venue m which is strictly greater than 0 and strictly lower than 1, for $i = 1, 2$ and $m = D, S$.

• *Conjecture 1: the best-quoting market-maker across venues is market-maker 1.* Under Conjecture 1, conditions (ii) and (iii') write $a_1^D Q_D + p_1^S Q_S - TC_1(Q_D + Q_S) \geq a_1^D Q_D - TC_1(Q_D)$ and $p_1^S Q_S < TC_2(Q_S)$. Condition (ii) rewrites $p_1^S Q_S \geq TC_1(Q_D + Q_S) - TC_1(Q_D)$. Combining with (iii'), we deduce that:

$$TC_1(Q_D + Q_S) < TC_1(Q_D) + TC_2(Q_S). \tag{A.1}$$

Straightforward computations show further that if Eq. (A.1) is verified, or equivalently $(I_1 - I_2 - Q_D) \times Q_S > 0$, then all conditions (i) to (iii') simultaneously hold, and Conjecture 1 is verified.

• *Conjecture 1a: the best-quoting market-maker across venues is market-maker 2.* In that case, conditions (i) and (i') rewrite $a_2^D Q_D + p_2^S Q_S \geq TC_2(Q_D + Q_S)$ and $a_1^D Q_D + p_1^S Q_S < TC_1(Q_D + Q_S)$. Given that market-maker 2 is the best-quoter, we obtain $a_2^D Q_D < a_1^D Q_D$ and $p_2^S Q_S < p_1^S Q_S$.[33] However, recall that $I_1 > I_2$ or, equivalently, $TC_1(Q_D + Q_S) < TC_2(Q_D + Q_S)$. Therefore, condition (i) cannot hold in that case and Conjecture 1a is not verified.

Case 2 ("Specialization"). We now look for the necessary conditions to be simultaneously filled to guarantee the existence of an equilibrium in which each liquidity shock is absorbed by a different market-maker.

There exists an equilibrium such that market-maker i posts the lowest ask price a_i^D in venue D and the opponent $-i$ posts the lowest ask (resp. highest bid) price p_i^S in venue S when $Q_S > 0$ (resp. $Q_S < 0$) if and only if:

(I) trading Q_D is profitable for market-maker i (i.e., $v_i(Q_D) \geq 0$), and (I') trading Q_S is profitable for market-maker $-i$ (i.e., $v_{-i}(Q_S) \geq 0$).

(II) market-maker i is better off trading Q_D rather than Q_S (i.e., $v_i(Q_D) > v_i(Q_S)$) and (III') market-maker $-i$ is better off trading Q_S rather than Q_D (i.e., $v_{-i}(Q_S) > v_{-i}(Q_D)$);

(III) market-maker i is better off trading Q_D only rather than $Q_D + Q_S$ (i.e., $v_i(Q_D) > v_i(Q_D + Q_S)$) and (II') market-maker $-i$ is better off trading Q_S only rather than $Q_D + Q_S$ (i.e., $v_{-i}(Q_S) > v_{-i}(Q_D + Q_S)$) ;

These conditions may be rewritten as follows:

$$\begin{aligned}
\text{I} &: a_i^D Q_D - TC_i(Q_D) \geq 0,\\
\text{I '} &: p_{-i}^S Q_S - TC_{-i}(Q_S) \geq 0,\\
\text{II} &: a_i^D Q_D - TC_i(Q_D) > p_i^S Q_S - TC_i(Q_S),\\
\text{II'} &: p_{-i}^S Q_S - TC_{-i}(Q_S) > a_{-i}^D Q_D - TC_{-i}(Q_D),\\
\text{III} &: a_i^D Q_D - TC_i(Q_D) > a_i^D Q_D + p_i^S Q_S - TC_i(Q_D + Q_S),\\
\text{III'} &: p_{-i}^S Q_S - TC_{-i}(Q_S) > a_{-i}^D Q_D + p_{-i}^S Q_S - TC_{-i}(Q_D + Q_S).
\end{aligned}$$

• *Conjecture 2: market-maker 1 trades Q_D and market-maker 2 trades Q_S.* Under Conjecture 2 and based on condition (III), we get $TC_1(Q_D + Q_S) - TC_1(Q_D) > p_1^S Q_S$. In case $Q_S > 0$, we know that $p_1^S > p_2^S$ and, using condition I', we get $TC_1(Q_D + Q_S) - TC_1(Q_D) > p_1^S Q_S > p_2^S Q_S > TC_2(Q_S)$. In case $Q_S < 0$, we get $-p_2^S Q_S \geq -p_1^S Q_S$, or using I' and III, we get $-TC_2(Q_S) > -p_2^S Q_S \geq -p_1^S Q_S > TC_1(Q_D) - TC_1(Q_D + Q_S)$. We thus obtain that:

$$TC_1(Q_D + Q_S) > TC_1(Q_D) + TC_2(Q_S) \tag{A.2}$$

Straightforward computations show that if Eq. (A.2) is verified then the set of conditions I to III' hold simultaneously and Conjecture 2 is verified.

• *Conjecture 2a: market-maker 1 trades Q_S and market-maker 2 trades Q_D.* Given that $I_1 < I_2$, straightforward computations lead to the following inequality:

$$TC_1(Q_D) + TC_2(Q_S) < TC_2(Q_D) + TC_1(Q_S). \tag{A.3}$$

[33] If $Q_S > 0$, $a_2^S < a_1^S$ and thus $a_2^S Q_S < a_1^S Q_S$. If $Q_S < 0$, $b_2^S > b_1^S$ and thus $b_2^S Q_S < b_1^S Q_S$. We thus can write $p_2^S Q_S < p_1^S Q_S$ for any sign of Q_S.

Under Conjecture 2a, we have $a_1^D Q_D > a_2^D Q_D$ and $p_2^S Q_S > p_1^S Q_S$. Combining Conjecture 2a with Inequality (A.3), we obtain $a_1^D Q_D + p_2^S Q_S - TC_1(Q_D) - TC_2(Q_S) > a_2^D Q_D + p_1^S Q_S - TC_2(Q_D) - TC_1(Q_S)$, which contradicts conditions II and II' combined. Conjecture 2a is thus not verified. ■

A.3 Proof of Proposition 1

From Lemma 1, we know that we must consider two cases according to the sign of $TC_1(Q_D + Q_S) - (TC_1(Q_D) + TC_2(Q_S))$, or, equivalently, of $(I_1 - I_2 - Q_D) \times Q_S$.

Case 1. Suppose that $(I_1 - I_2 - Q_D) \times Q_S > 0$ **("Virtual consolidation").** In that case, we know that market-maker 1 posts the best prices across venues (Lemma 1). We now have to consider two sub-cases according to the sign of Q_S.

Case 1.1. Suppose that $Q_S > 0$. Following Condition (A.1), we must have $I_1 - I_2 > Q_D$. Market-maker 1 posts the lowest selling price both in venue D and S. The ask prices a_1^D and a_1^S are the maximum prices that satisfy the set of conditions i to iii' (Lemma 1). Combining conditions (ii') and (iii) and conditions (ii) and (iii') and using reservation prices, we get:

$$\text{ii' and iii} : r_1(Q_D) + \rho\sigma^2 Q_S \leq a_1^D < r_2(Q_D),$$
$$\text{ii and iii'} : r_1(Q_S) + \rho\sigma^2 Q_D \leq a_1^S < r_2(Q_S).$$

From the two first inequalities, natural candidates for the equilibrium are $(a_1^D)^* = r_2(Q_D) - \varepsilon$ and $(a_1^S)^* = r_2(Q_S) - \varepsilon$, as they are the maximum prices that satisfy conditions ii and iii, ii' and iii'. Straightforward computations show that they also satisfy conditions i and i' described above (details are omitted for brevity).

Case 1.2. Suppose that $Q_S < 0$. In that case, we must have $I_1 - I_2 < Q_D$ to satisfy Condition (A.1). Market-maker 1 thus posts the lowest selling price in venue D and the highest bid price in venue S. The ask price a_1^D and the bid price b_1^S are such that they must satisfy the set of conditions (ii) to (iii') that we rewrite as follows:

$$\text{ii' and iii} : r_1(Q_D) + \rho\sigma^2(Q_S) \leq a_1^D < r_2(Q_D),$$
$$\text{ii and iii'} : r_2(Q_S) < b_1^S \leq r_1(Q_S) + \rho\sigma^2 Q_D.$$

The natural candidates for the equilibrium are $a_1^D = r_2(Q_D) - \varepsilon$ and $b_1^S = r_2(Q_S) + \varepsilon$. These equilibrium prices must satisfy the following inequality $a_1^D Q_D + b_1^S Q_S < (a_2^D Q_D + b_2^S Q_S \leqslant) TC_2(Q_D + Q_S)$ (condition (i')). It is however not the case, implying that this constraint is binding and equilibrium prices must be such that:

$$(a_1^D)^* = r_2(Q_D + Q_S)\frac{(Q_D + Q_S)}{Q_D} + (b_1^S)^* \frac{(-Q_S)}{Q_D} - \varepsilon. \tag{A.4}$$

First, using the expression of $(a_1^D)^*$ defined in Eq. (A.4) in market-maker 1's trading profit, we obtain $v_1(Q_D + Q_S) = \rho\sigma^2(I_1 - I_2)(Q_D + Q_S)$. This expression does not depend on equilibrium prices. Consequently, there exists a continuum of prices that may sustain the equilibrium. Second, using $(a_1^D)^*$ defined in Eq. (A.4) in conditions (ii') and (iii) combined, we get that $(b_1^S)^*$ must satisfy:

$$\text{ii' and iii} : r_2(Q_S) - \rho\sigma^2(I_1 - I_2)\frac{Q_D}{-Q_S} \leq (b_1^S)^* < r_2(Q_S) + \rho\sigma^2 Q_D.$$

We also know from conditions (ii) and (iii') combined that $(b_1^S)^*$ is such that:

$$\text{ii and iii'} : r_2(Q_S) < (b_1^S)^* \leq r_1(Q_S) + \rho\sigma^2 Q_D.$$

Since $I_1 > I_2$, we however have $r_2(Q_S) - \rho\sigma^2(I_1 - I_2)\frac{Q_D}{-Q_S} < r_2(Q_S)$ and $r_1(Q_S) + \rho\sigma^2 Q_D < r_2(Q_S) + \rho\sigma^2 Q_D$. The second inequality defined by (ii) and (iii') combined is constraining both the minimum and the maximum possible bid price in venue S. Within all equilibria defined by $(a_1^D)^*$ in Eq. (A.4) and by $(b_1^S)^* \in (r_2(Q_S) + \epsilon, r_1(Q_S) + \rho\sigma^2 Q_D + \epsilon]$ we select the only equilibrium such that prices are continuous at $I_1 - I_2 = Q_D$, that is, $(a_1^D)^* = r_2(Q_D) + \rho\sigma^2(Q_S) - \epsilon \equiv \hat{r}_2(Q_D) - \epsilon$, from which we deduce that $(b_2^S)^* = r_2(Q_S) + \epsilon$.

Case 2. Suppose that $(I_1 - I_2 - Q_D) \times Q_S < 0$ **("Specialization").** From Lemma 1, we know that market-maker 1 posts the best price in venue D while market-maker 2 posts the best price in venue S. We now have to consider two sub-cases according to the sign of Q_S.

Case 2.1. Suppose that $Q_S > 0$**.** In that case, we must have $I_1 - I_2 < Q_D$ to satisfy Condition (A.1). The ask price a_1^D posted by market-maker 1 and the ask price a_2^S posted by market-maker 2 are such that they must satisfy the set of conditions I to III', from which we deduce that:

$$\text{I and III'} : r_1(Q_D) \leq a_1^D < a_2^D < r_2(Q_D) + \rho\sigma^2 Q_S,$$
$$\text{I' and III} : r_2(Q_S) \leq a_2^S < a_1^S < r_1(Q_S) + \rho\sigma^2 Q_D.$$

The candidates for the equilibrium are $a_1^D = r_2(Q_D) + \rho\sigma^2 Q_S - \varepsilon$ and $a_2^S = r_1(Q_S) + \rho\sigma^2 Q_D - \varepsilon$. These equilibrium prices must satisfy the following inequality $a_2^S Q_S - a_1^D Q_D (> a_2^S Q_S - a_2^D Q_D) > r_2(Q_S)Q_S - r_2(Q_D)Q_D$ (condition (II')). It is however not the case, implying that this constraint is binding and equilibrium prices must be such that:

$$(a_1^D)^* = r_2(Q_D) + ((a_2^S)^* - r_2(Q_S))\frac{Q_S}{Q_D} - \varepsilon. \tag{A.5}$$

First, if $(a_1^D)^*$ defined in Eq. (A.5), then condition II always holds (given that $(I_1 - I_2)(Q_D - Q_S) > 0$). Second, using $(a_1^D)^*$ defined in Eq. (A.5) in conditions I and III' and I' and III combined, we get that $(a_2^S)^*$ must satisfy the following inequalities:

$$\text{I and III'} : r_2(Q_S) + (r_1(Q_D) - r_2(Q_D))\frac{Q_D}{Q_S} \leq (a_2^S)^* < r_2(Q_S) + \rho\sigma^2 Q_D,$$
$$\text{I' and III} : r_2(Q_S) \leq (a_2^S)^* < r_1(Q_S) + \rho\sigma^2 Q_D.$$

Straightforward computations show that conditions I' and III combined is constraining the set of possible prices $(a_2^S)^*$. Third, we compute market-makers' equilibrium profits and show that, in that case, the trading profit of market-maker 2 writes: $v_2(Q_S) = ((a_2^S)^* - r_2(Q_S))Q_S$. Using the expression of $(a_2^D)^*$ defined in Eq. (A.5), we then obtain that the trading profit of market-maker 1 writes:

$$v_1(Q_D) = \left(r_2(Q_D) + ((a_2^S)^* - r_2(Q_S))\frac{Q_S}{Q_D} - r_1(Q_D)\right) Q_D.$$

We observe that market-makers' profits are both strictly increasing in $(a_2^S)^*$. Consequently, market-makers' reaction functions are such that the best ask price in venue S is the highest possible one. From conditions I and III' combined, we deduce that $(a_S)^*$ is such that:

$$(a_2^S)^* = r_1(Q_S) + \rho\sigma^2 Q_D - \varepsilon, \text{ or } (a_2^S)^* = \hat{r}_1(Q_S) - \varepsilon, \tag{A.6}$$

from which we deduce that:

$$(a_1^D)^* = \hat{r}_2(Q_D) - \rho\sigma^2 Q_S \times \eta - \varepsilon, \tag{A.7}$$

where $\eta = \frac{(I_1 - I_2)}{Q_D}$.

Consequently, there exists a unique equilibrium such that market-maker 1 posts $(a_1^D)^*$ (defined in Eq. (A.7)) and trades Q_D while market-maker 2 posts the best ask price equal to $(a_2^S)^*$ (defined in Eq. (A.6)) and trades Q_S.

Case 2.2. Suppose that $Q_S < 0$**.** In that case, we have $I_1 - I_2 > Q_D$ (Condition (A.1)). Market-maker 1 posts the best ask price in D while market-maker 2 posts the best bid price in S. The ask price a_1^D in venue D and the bid price b_2^S in venue S are respectively the maximum and the minimum prices that satisfy the set of conditions I to III'. Combining Condition (II) and (III) and Condition (II') and (III'), we get:

$$\text{II and III} : r_1(Q_D) \leq a_1^D < r_2(Q_D) + \rho\sigma^2 Q_S,$$
$$\text{II' and III'} : r_1(Q_S) + \rho\sigma^2 Q_D < b_2^S \leq r_2(Q_S).$$

From the two first inequalities, $a_1^D = r_2(Q_D) - \rho\sigma^2(-Q_S) - \varepsilon$ and $b_2^S = r_1(Q_S) + \rho\sigma^2 Q_D + \varepsilon$ are natural candidates for the equilibrium. Straightforward computations show that they also satisfy conditions I and I'.■

A.4 Proof of Proposition 2

We decompose the proof into two results, depending on the sign of Q_S.

Notations. For ease of computation in the proof, we use the following notations $q_m = Q_m$ for a net-buying order flow and $q_m = -Q_m$ for a net-selling order flow ($m = S, D$). Let us also define $v_d = \mu - \rho\sigma^2 I_d$, $v_u = \mu - \rho\sigma^2 I_u$, $x = \mu - \rho\sigma^2 I_1$ and $y = \mu - \rho\sigma^2 I_2$. The support of the uniform distribution function of x and y simplifies to $[v_u, v_d]$. We also note $d = \rho\sigma^2 q_D$ and $s = \rho\sigma^2 q_S$. Finally, let $a^{m,+}$ (resp. $a^{m,-}$) be the best ask price of venue m when liquidity demands have the same sign (resp. opposite sign) across venues.

Result 1 *Suppose that shocks have the same sign (with probability γ). Then, the expected ask prices quoted in the venues D and S are equal to:*

$$E\left(\underline{a}^{m,+}\right) = \mu - \rho\sigma^2\frac{2I_d + I_u}{3} + \frac{\rho\sigma^2 q_m}{2} + \rho\sigma^2 q_{-m}\left(\frac{q_D}{I_u - I_d} - \frac{1}{3}\left(\frac{q_D}{I_u - I_d}\right)^2\right), m = S, D. \tag{A.8}$$

Proof. We first compute the expected ask that prevails in venue D. By definition,

$$E\left(\underline{a}^{D,+}\right) = E\left(\min\left(a_1^D, a_2^D\right)\mathbb{1}_{Q_D>0}\mathbb{1}_{Q_S>0}\right).$$

Given Proposition 1, the notations defined above, and the symmetry of our hypotheses, the latter equation writes:

$$\begin{aligned} E\left(\underline{a}^{D,+}\right) &= \frac{2}{(v_d - v_u)^2}\Bigg[\int_{v_u}^{v_d - d}\int_{x+d}^{v_d}(y + \frac{d}{2})dydx + \int_{v_u}^{v_d}\int_{x}^{v_d}\left(y + \frac{d}{2} + s\left(\frac{d-(y-x)}{d}\right)\right)dydx \\ &- \int_{v_u}^{v_d-d}\int_{x+d}^{v_d}\left(y + \frac{d}{2} + s\left(\frac{d-(y-x)}{d}\right)\right)dydx\Bigg]. \end{aligned} \tag{A.9}$$

We now turn to the expected ask prevailing in venue S using a similar reasoning. The expression writes:

$$\begin{aligned} E\left(\underline{a}^{S,+}\right) &= E\left(\min\left(a_1^S, a_2^S\right)\mathbb{1}_{Q_D>0}\mathbb{1}_{Q_S>0}\right) \\ &= \frac{2}{(v_d - v_u)^2}\Bigg[\int_{v_u}^{v_d - d}\int_{x+d}^{v_d}(y + \frac{s}{2})dydx + \int_{v_u}^{v_d}\int_{x}^{v_d}\left(x + \frac{s}{2} + d\right)dydx \\ &- \int_{v_u}^{v_d-d}\int_{x+d}^{v_d}\left(x + \frac{s}{2} + d\right)dydx\Bigg]. \end{aligned} \tag{A.10}$$

Computations based on Eq. (A.9) and on Eq. (A.10) yield the expressions given in Eq. (A.8) for $m = D$ and $m = S$ respectively. Q.E.D.

Result 2 *Suppose that shocks have opposite signs (with probability $1-\gamma$), then the expected ask prices in venues D and S respectively write:*

$$E\left(\underline{a}^{D,-}\right) = \mu - \rho\sigma^2\frac{2I_d + I_u}{3} + \frac{\rho\sigma^2 q_D}{2} - \rho\sigma^2 q_S, \tag{A.11}$$

$$E\left(\underline{a}^{S,-}\right) = \mu - \rho\sigma^2\frac{2I_d + I_u}{3} + \frac{\rho\sigma^2 q_S}{2} - \rho\sigma^2 q_D + \frac{(q_D)^2}{(I_u - I_d)} - \frac{(q_D)^3}{3\left(I_u - I_d\right)^2}. \tag{A.12}$$

Proof. We first compute the expected best ask prevailing in venue D (considering a sell shock in venue S):

$$E\left(\underline{a}^{D,-}\right) = E\left(\min\left(a_1^D, a_2^D\right)\mathbb{1}_{Q_D>0}\mathbb{1}_{Q_S<0}\right),$$

which rewrites:

$$E\left(\underline{a}^{D,-}\right) = \frac{2}{\left(v_d - v_u\right)^2}\Big(\int_{v_u}^{v_d-d}\int_{v_u}^{x+d}(y+\frac{d}{2}-s)dydx + \int_{v_u}^{v_d}\int_{x}^{v_d}(y+\frac{d}{2}-s)dydx - \int_{v_u}^{v_d-d}\int_{x+d}^{v_d}(y+\frac{d}{2}-s)dydx\Big). \quad (A.13)$$

Eq. (A.11) immediately follows.
Symmetrically, the expected best ask prevailing in market S (considering now a sell shock in venue D) writes:

$$E(\underline{a}^{S,-}) = \frac{2}{(v_d - v_u)^2}\Big(\int_{v_u-d}^{v_d}\int_{v_u}^{x+d}(x+\frac{s}{2}+d)dydx + \int_{v_u}^{v_d}\int_{v_u}^{x}(y+\frac{s}{2})dydx - \int_{v_u-d}^{v_d}\int_{v_u}^{x+d}(y+\frac{s}{2})dydx\Big). \quad (A.14)$$

Computations yield Eq. (A.12). Q.E.D.
Let us define the half-spread as $s^m = a^m - \mu$ and $\phi_m = \frac{q_m}{I_u - I_d}$. Proposition 2 is then obtained from Results 1 and 2 considering the extensive form of the game represented in Figure 2. ■

A.5 Proof of Corollary 1

Remind that $\underline{a}^c$ denotes the lowest ask price in a centralized market. From Ho and Stoll (1983), we know that:

$$E\left(\underline{a}^c\right) = \mu - \rho\sigma^2\frac{2I_d + I_u}{3} + \frac{\rho\sigma^2(q_m + q_{-m})}{2}. \quad (A.15)$$

Using Eq. (A.8), (A.11) and (A.20) and the symmetry of the game, we deduce that the difference in expected transactions costs between a fragmented and a centralized market is:

$$\begin{aligned}\Delta E(TTrC) &= \gamma\left(E\left(\underline{a}^{D,+}\right)q_D + E\left(\underline{a}^{S,+}\right)q_S - E\left(\underline{a}^c\right)(q_D + q_S)\right) \\ &+ (1-\gamma)\left(E\left(\underline{a}^{D,-}\right)q_D - E\left(\overline{b}^{S,-}\right)q_S - E\left(\underline{a}^c\right)(q_D - q_S)\right).\end{aligned}$$

After straightforward computations the latter expression is equal to:

$$\Delta E(TTrC) = \rho\sigma^2 q_S\left(I_u - I_d\right)\left(-\frac{(\gamma+1)}{3}\right)P_\gamma(\phi_D), \quad (A.16)$$

where $P_\gamma(x) = x^3 - 3x^2 + \frac{3}{(\gamma+1)}x + \frac{(\gamma-1)}{(\gamma+1)}$ for $x \in [0,1]$, and $\phi_D = \frac{q_D}{I_u - I_d}$.
To investigate whether expected transaction costs are larger or smaller in a centralized market, let us analyze the sign of the cubic polynomial P_γ. First, note that:

$$P'_\gamma(x) = 3x^2 - 6x + \frac{3}{(1+\gamma)} = 3\left(x - \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)\right)\left(x - \left(1 + \sqrt{\frac{\gamma}{1+\gamma}}\right)\right).$$

Given that $x \in [0,1]$, then $x - \left(1 + \sqrt{\frac{\gamma}{1+\gamma}}\right) < 0$, and the sign of $P'_\gamma(x)$ only depends on the sign of $\left(x - \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)\right)$. P_γ is increasing if $x < \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)$ and is decreasing if $x > \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)$. Thus, the local maximum is $P_\gamma(1 - \sqrt{\frac{\gamma}{1+\gamma}}) = \frac{\gamma\left(-1+2\sqrt{\frac{\gamma}{1+\gamma}}\right)}{1+\gamma}$.

• Consider the case where $\gamma \leq \frac{1}{3}$. Straightforward computations show that $P_\gamma(1 - \sqrt{\frac{\gamma}{1+\gamma}}) \leq 0$ (with $P_\gamma(1 - \sqrt{\frac{\gamma}{1+\gamma}}) = 0$ if $\gamma = \frac{1}{3}$). We therefore deduce that $P_\gamma \leq 0$, i.e., $\Delta E(TTrC) \geqslant 0$ if $\gamma \leq \frac{1}{3}$.

• Consider now the case where $\gamma > \frac{1}{3}$. We can show that $P_\gamma > 0$, or, equivalently, $\Delta E(TTrC) < 0$ iff $x \in [\Phi^1_\gamma, \Phi^2_\gamma]$ where $P_\gamma(\Phi^1_\gamma) = 0 = P_\gamma(\Phi^2_\gamma)$. Note that if $\gamma = 1$, then it is direct to show that $P_1 > 0$ if

$x \in [0, \frac{(3-\sqrt{3})}{2}]$, or equivalently, $\Delta E(TTrC) < 0$ iff $\phi_D < \frac{(3-\sqrt{3})}{2}$.■

A.6 Proof of Proposition 3

By definition, $Cov(s^D, s^S) = \gamma Cov(\underline{a}^{D,+} - \mu, \underline{a}^{S,+} - \mu) + (1-\gamma) Cov(\underline{a}^{D,-} - \mu, \mu - \overline{b}^{S,-}) = \gamma Cov(\underline{a}^{D,+}, \underline{a}^{S,+}) - (1-\gamma) Cov(\underline{a}^{D,-}, \overline{b}^{S,-})$. We thus decompose the proof into two results, depending on the sign of shocks across venues (similar or opposite).

Result 3 *Suppose that shocks have the same sign (with probability γ). The covariance between the ask price in venue D and the one in venue S is equal to:*

$$\frac{Cov(\underline{a}^{D,+}, \underline{a}^{S,+})}{(\rho\sigma^2)^2 (I_u - I_d)^2} = \frac{1}{18} - \phi_D \left(-\frac{\phi_D - \phi_S}{6} + \frac{2(\phi_S - \phi_D)}{9}\phi_D + \frac{15\phi_S - \phi_D}{12}\phi_D^2 + \frac{2\phi_S}{3}\phi_D^3 + \frac{\phi_S}{9}\phi_D^4 \right), \tag{A.17}$$

where $\phi_m = \frac{q_m}{(I_u - I_d)}$, $m = D, S$.

Proof. By definition, $E\left(\underline{a}^{D,+}\underline{a}^{S,+}\right) = E\left(\min\left(a_1^D, a_2^D\right) \times \min\left(a_1^S, a_2^S\right) \mathbb{1}_{Q_D>0}\mathbb{1}_{Q_S>0}\right)$. Using Proposition 1, and notations defined above, we get:

$$\begin{aligned} E\left(\underline{a}^{D,+}\underline{a}^{S,+}\right) = \frac{2}{(v_d - v_u)^2}\Bigg[&\int_{v_u}^{v_d - d}\int_{v_u}^{v_d - d} (y + \frac{s}{2})(y + \frac{d}{2}) dy dx \\ &+ \int_{v_u}^{v_d}\int_{x}^{v_d} (x + \frac{s}{2} + d)(y + \frac{d}{2} + s\left(\frac{d - (y - x)}{d}\right)) dy dx \\ &- \int_{v_u}^{v_d - d}\int_{x+d}^{v_d} (x + \frac{s}{2} + d)(y + \frac{d}{2} + s\left(\frac{d - (y - x)}{d}\right)) \Bigg]. \end{aligned} \tag{A.18}$$

To compute $Cov(\underline{a}^{D,+}, \underline{a}^{S,+}) = E(\underline{a}^{D,+}\underline{a}^{S,+}) - E(\underline{a}^{D,+})E(\underline{a}^{S,+})$, we use the expression above and Result 1 for expressions of $E\left(\underline{a}^{D,+}\right)$ and $E\left(\underline{a}^{S,+}\right)$. Computations yield Eq. (A.17). Q.E.D.

Result 4 *Suppose that shocks have opposite signs $(1-\gamma)$. The covariance between the best price in venue D and the one in venue S writes:*

$$\frac{Cov\left(\underline{a}^{D,-}, \overline{b}^{S,-}\right)}{(\rho\sigma^2)^2 (I_u - I_d)^2} = \frac{1}{36} + \frac{(\phi_D)^2}{36}\left(3(\phi_D)^2 - 8\phi_D + 6\right). \tag{A.19}$$

Proof. If a sell shock hits venue S, the expected best bid in venue S is such that $E(\overline{b}^{S,-}) = E(max(b_1^S, b_2^S)\mathbb{1}_{Q_D>0}\mathbb{1}_{Q_S<0})$, or:

$$\begin{aligned} E(\overline{b}^{S,-}) = \frac{2}{(v_d - v_u)^2}\Big(&\int_{v_u}^{v_d - d}\int_{x+d}^{v_d} (x + \frac{s}{2} + d) dy dx + \int_{v_u}^{v_d}\int_{x}^{v_d} (y + \frac{s}{2}) dy dx \\ &- \int_{v_u}^{v_d - d}\int_{x+d}^{v_d} (y + \frac{s}{2}) dy dx \Big) \end{aligned} \tag{A.20}$$

When a buy shock hits venue D, the expected best ask price of venue D is thus described by Eq. (A.11). Then $E(\underline{a}^{D,-}\overline{b}^{S,-})$ writes:

$$\begin{aligned} E(\underline{a}^{D,-}\overline{b}^{S,-}) = \frac{2}{(v_d - v_u)^2}\Big[&\int_{v_u}^{v_d - d}\int_{x+d}^{v_d} (y + \frac{d}{2} - s)(x + \frac{s}{2} + d) dy dx \\ &+ \int_{v_u}^{v_d}\int_{x}^{v_d} (y + \frac{d}{2} - s)(y + \frac{s}{2}) dy dx - \int_{v_u}^{v_d - d}\int_{x+d}^{v_d} (y + \frac{d}{2} - s)(y + \frac{s}{2}) dy dx \Big]. \end{aligned} \tag{A.21}$$

Using Equations (A.11), (A.20) and (A.21), we can deduce the expression of $Cov(\underline{a}^{D,-}, \overline{b}^{S,-})$ described in Eq. (A.19). Q.E.D.

From Results 3 and 4 and considering the extensive form of the game represented in Figure 2, we deduce that spreads co-vary jointly as follows:

$$cov(s^D, s^S) = \lambda\big(\rho\sigma^2(I_u - I_d)\big)^2\big(\gamma \times g_{\phi_D}(\phi_S) + a_{\phi_D}\big) \tag{A.22}$$

where a_{ϕ_D} and g_{ϕ_D} such that: $a_{\phi_D} = \frac{-1}{36} - (\phi_D)^2(\frac{1}{6} - \frac{2}{9}\phi_D)$ and

$$g_{\phi_D}(x) = \frac{3}{36} - \frac{(\phi_D)^4}{12} - x\phi_D\left(\frac{(\phi_D)^4}{9} - \frac{2(\phi_D)^3}{3} + \frac{5(\phi_D)^2}{4} - \frac{8\phi_D}{9} + \frac{1}{6}\right). \tag{A.23}$$

It is straightforward to show that g_{ϕ_D} is positive for any ϕ_D. We thus deduce that the covariance $cov(s^D, s^S)$ is an increasing function of γ.■

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Online Appendix to "Fragmentation and Strategic Market-Making"

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The aim of this online Appendix is to present a number of additional results and robustness checks. Appendix A analyzes quotes posted by non-strategic market-makers, and shows that the "ultra-competitive effect" uncovered in the baseline model results from the strategic behavior of market-makers and not from a pure inventory management effect. Appendix B investigates whether a fragmented market with multi-venue market-makers increases the extent to which market-makers can share risks. Appendix C aims at relaxing the hypothesis that the market is exogenously fragmented, and shows that, even in the case of an endogenous fragmentation, the market remains fragmented. Appendix D investigates whether risk-averse market-makers would prefer trading and sharing risks together in a pre-trading stage. Finally, Appendix E shows that our model does not allow any trade-through. Prices are different across trading venues because of difference in orders size or direction.

Contents

A Non-strategic quotes in a fragmented market

Our paper assume that market-makers behave strategically within and across trading venues. Most of the literature however analyzes market fragmentation by assuming that liquidity suppliers behave competitively, setting prices such that a zero-profit condition holds. This appendix analyzes prices posted by competitive risk-averse market makers in our two-market setting. It allows us to better understand the impact of assuming *strategic* multi-venue market-makers. It also allows us to show that the "ultra-competitive effect" obtained in the model resulting from the strategic behavior of risk-averse market-makers is not obtained under the same conditions in a non-strategic inventory management model.

Proposition A.1 *Assume that* $I_1 > I_2$ *and* $Q_D + Q_S > 0$*, and that market-makers behave competitively. Then they quote their true value for the asset, i.e., their own reservation price, that is:*

1. If $(I_1 - I_2 - Q_D)Q_S > 0$*, then market-maker 1, with a longer position, posts the best prices across venues, that is:*

$$(p_1^m)^{NS} = r_1(Q_D + Q_S) \quad \text{for } m = D, S \tag{1}$$

2. If $(I_1 - I_2 - Q_D)Q_S \leq 0$*, the longer market-maker posts the best price in the dominant market while the shorter market-maker posts the best price in the satellite market, that is:*

$$((a_1^D)^{NS}, (p_2^S)^{NS}) = (r_1(Q_D), r_2(Q_S)) \tag{2}$$

where p_i^m *is a selling price when* Q_m *is a buy demand, and a bid price when* Q_m *is a sell demand (*$i = 1, 2,$ *and* $m = D, S$*).*

In a multi-venue environment, when intermediaries are competitive, best prices sometimes differ across venues for two reasons. First, when $(I_1 - I_2 - Q_D)Q_S \leq 0$, which is equivalent to $TC_1(Q_D + Q_S) \geq TC_1(Q_D) + TC_2(Q_S)$, market-maker 1 is capacity-constrained and cannot absorb both shocks. She is thus constrained to absorb the most efficient shock (in terms of risk sharing), which is the larger buy demand Q_D sent to the dominant venue (see Lemma 1 in the baseline model). She thus posts her true value for executing Q_D, while market-maker 2 executes the shock in the satellite market at his reservation price for this liquidity demand Q_S. Second, because market-makers' reservation quotes reflect the price impact of trades of different size ($|Q_D| > |Q_S|$), when market-makers are not strategic, they post prices that reflect their true value for the magnitude of orders to execute, which sometimes differ (alternatively $Q_D + Q_S$ or Q_D for market-maker 1, or Q_S for market-maker 2).

Let us now analyze the impact in terms of total trading costs. We assume also that market-makers behave non strategically in a centralized market, and thus post their true value for the asset. Market-maker 1 executes the net order flow at her reservation price $r_1(Q_D + Q_S)(< r_2(Q_D + Q_S))$. According to Lemma 1 in the baseline model, we have two cases to consider:

- If $(I_1 - I_2 - Q_D)Q_S \leq 0$, then total trading costs are the same. Fragmentation is thus innocuous in this case.
- If $(I_1 - I_2 - Q_D)Q_S \leq 0$, then the difference in total trading costs write:

$$r_1(Q_D)Q_D + r_2(Q_S)Q_S - r_1(Q_D + Q_S) \times (Q_D + Q_S) = \rho \times \sigma^2(I_1 - I_2 - Q_D) \times Q_S < 0. \quad (3)$$

  We thus deduce that market fragmentation is good for total trading costs in this case.

In a two-venue setting with competitive market-makers, total trading costs are lower than in a centralized market. It is driven by a better risk sharing in the case market-maker 1 is capacity-constrained. This outcome is opposite to that obtained in our two-venue strategic duopoly model. Recall that when market-maker 1 is capacity-constrained, market-makers price high, by posting their "stay-at-home" price which takes into account their monopolist situation in their "home" venue. A better risk sharing leads to less competitive prices in our baseline model (see Appendix B for a more formal proof on risk sharing).

In sum, the "intense competition" effect results from low divergence in inventories when market-makers behave non-strategically, whereas this effect is obtained when divergence in inventories is high if intermediaries behave strategically. Our main empirical result contained in Table 3 in the main model corroborates strategic inventory management.

B Risk-sharing efficiency in a fragmented market

This Appendix explores the impact of the possibility to absorb the preferred shock (what we termed in the baseline model as the shock in their "home" venue) on risk sharing among market-makers. It is worth noticing that, when intermediaries specialize in their "home" venue, they obtain a better allocation of risk compared to the centralized market, as shown in the following corollary.

Corollary B.1 *A fragmented market generates a more efficient outcome in risk sharing among market-makers than a centralized market in the sense that market-makers bear lower aggregate security risk.*

Proof. In our set up (equal risk aversion and identical pre-trade inventory distribution), we can measure intermediaries' aggregate post-trade risk by the sum of the variance of their post-trade wealths (Yin, 2005).

1. In a centralized market, the longer market-maker executes the net order flow. The aggregate post-trade risk, denoted by $(\sigma^2_{agg})^c$, is thus equal to:

$$(\sigma^2_{agg})^c = Var((I_1 - Q_D - Q_S)\tilde{v}) + Var(I_2 \times \tilde{v}). \tag{4}$$

2. In a fragmented market, post-trade allocations depend on the sign of $(I_1 - I_2 - Q_D\,)\,Q_S$ (See Lemma 1 in the baseline model).

   • If $(I_1 - I_2 - Q_D)\,Q_S > 0$, the aggregate post-trade risk is equal to that in a centralized market, since the longer market-maker consolidates the global order flow:

$$(\sigma^2_{agg})^{cons} = Var((I_1 - Q_D - Q_S)\tilde{v}) + Var(I_2 \times \tilde{v}) = (\sigma^2_{agg})^c.$$

   • If $(I_1 - I_2 - Q_D)\,Q_S \leq 0$, each shock is absorbed by a different market-maker and the aggregate post-trade risk is equal to:

$$(\sigma^2_{agg})^{frag} = Var((I_1 - Q_D)\tilde{v}) + Var((I_2 - Q_S)\tilde{v}). \tag{5}$$

   Then, subtracting Eq. (5) from Eq. (4) is equal to $(\sigma^2_{agg})^{frag} - (\sigma^2_{agg})^c = 2Q_S(I_1 - I_2 - Q_D) < 0$, which is negative in the case considered here.■

The intuition is as follows: in a centralized market, orders are crossed when they are of opposite direction. This implies that, if $Q_S < 0$, market-makers only absorb the remaining order imbalance of $Q_D + Q_S < Q_D$. In a multiple-venue setting, orders cannot be crossed, market-makers are however able to choose to execute only trades with a desirable impact on their inventory position. In the case $Q_S < 0$, market-maker 1 chooses to absorb only Q_D when she is very long ($I_1 - I_1 - Q_D > 0$), while the shorter market-maker absorb the shock in S, which results in a better risk sharing than in the centralized case. The better allocation of risk does not however necessarily lead to more competitive prices as detailed in Proposition 1 in the baseline model. This result is in the spirit of the one obtained in Biais et al. (1998).

C Endogenous fragmentation of the total order flow

This Appendix extends the baseline model by assuming that a global liquidity demander has access to all venues simultaneously (through, for example, a smart order router). Let us assume that this liquidity demander must trade a given quantity denoted Q. He minimizes his total trading cost by optimally splitting orders across venues. Note that this section only extends the case in which shocks have exogenously the same sign in our baseline model. We also assume that market-maker 1 is longer than market-maker 2 and that Q is a buy order flow ($Q > 0$). Results for the case in which market-maker 2 is longer that market-maker 1 or for the case of a sell order are deduced by symmetry.

We consider that the global liquidity demander enjoys some private benefits denoted δ_m to trade in venue m. We assume that $\delta_D > \delta_S$, consistently with the dominant venue defined in the baseline model, and that $\delta_D - \delta_S < \rho\sigma^2 Q$.[1,2] The liquidity demander chooses the proportion α of the order flow routed to venue D (and $(1-\alpha)$ to venue S) so as to minimize his total trading cost.[3]

Based on assumptions in the baseline model, we suppose that the liquidity demander splits orders such that a larger demand is sent to the dominant market ($\alpha Q = Q_D \geq Q_S = (1-\alpha)Q$).[4] We thus investigates whether there exists an equilibrium when the liquidity trader optimally split orders across venues such that $\alpha \in [\frac{1}{2}; 1)$. In this interval, total trading costs write:

$$TTrC(\alpha) = [((a_1^D)^*(\alpha Q) - \delta_D - \mu)\alpha + ((a_i^S)^*((1-\alpha)Q) - \delta_S - \mu)(1-\alpha)] \times Q. \quad (6)$$

where $i = 1, 2$ depending on the divergence in inventories ($i = 1$ if $I_1 - I_2 > \alpha Q$, $i = 2$ otherwise, see Lemma 1 in the baseline model).

The following Proposition shows the existence and the characterization of an equilibrium α^*.

[1]Numerous studies (see Froot and Dabora, 1999; Foerster and Karolyi, 1999; or Gagnon and Karolyi, 2010, among others) document the existence of a domestic bias, due to investment barriers, e.g., regulatory barriers, taxes, or information constraints. In Europe, brokerage fees charged in 2013 to trade in a foreign country or trading venue are 15 to 40% higher than those charged to trade in a national exchange, but the situation was even worse back in 2007 (see documents on Fees and Commissions of various brokers from 2007 to 2013). Differences in private benefits might also capture differences in terms of maker/taker spreads.

[2]When $\delta_D - \delta_S \geq \rho\sigma^2 Q$, the private benefits of trading in venue D are so large that it is never optimal for investors to split the quantity to be traded across trading platforms.

[3]Because markets are transparent in our set up, we assume that liquidity demanders perfectly anticipate what the best bid and ask prices will be.

[4]A complete proof of the existence and characterization of all the equilibria is available on request.

Proposition C.1 *If $2\rho\sigma^2(I_1 - I_2) > (\delta_D - \delta_S)$, there exists an interior equilibrium α^*, such that it is optimal for the global liquidity demander to split orders across venues.*

Proof. We want to show that there exists an interior equilibrium, that is, an $\alpha^* \in [\frac{1}{2}, 1)$ that minimizes transaction costs $TTrC(.)$ described by Eq. (6).

• We first conjecture that there exists an equilibrium characterized by a high divergence in intermediaries' inventories, i.e., $I_1 - I_2 - \alpha Q > 0$, or, $\frac{1}{2} \leq \alpha < \frac{I_1 - I_2}{Q}$. The first order condition (FOC) yields:

$$\alpha^H = \frac{1}{2} + \frac{\delta_D - \delta_S}{2\rho\sigma^2 Q}.$$

The two conditions for an interior equilibrium $\alpha \in [\frac{1}{2}, 1)$ to exist are thus: i. a condition ensuring that our conjecture holds, i.e., $\alpha^H < \frac{I_1 - I_2}{Q}$, and ii. a condition ensuring that the equilibrium is interior, i.e., $\alpha^H < 1$. The latter always holds under our assumption $\delta_D - \delta_S < \rho\sigma^2 Q$. Condition i. rewrites as follows:

$$I_1 - I_2 > \frac{1}{2}\left(Q + \frac{\delta_D - \delta_S}{\rho\sigma^2}\right). \tag{7}$$

• We now conjecture that there exists an equilibrium characterized by a low divergence in intermediaries' inventories, i.e., $\alpha \geq \frac{I_1 - I_2}{Q}$. The FOC yields:

$$\alpha^L = \frac{1}{2} - \frac{\delta_D - \delta_S}{2\rho\sigma^2 Q} + \frac{(I_1 - I_2)}{Q}.$$

The three conditions for an interior equilibrium to exist are such that: (i) our conjecture must hold, i.e., $\alpha^L \geq \frac{I_1 - I_2}{Q}$; (ii) there exists an interior equilibrium, i.e., $\alpha^L < 1$; and (iii) $\alpha^L \geq \frac{1}{2}$. Condition (i) always holds under our assumption $\delta_D - \delta_S < \rho\sigma^2 Q$. Condition (ii) translates into $I_1 - I_2 < \frac{Q}{2} + \frac{\delta_D - \delta_S}{2\rho\sigma^2}$, which is the complement of the condition (7) above. Notice that if $I_1 - I_2 = \frac{Q}{2} + \frac{\delta_D - \delta_S}{2\rho\sigma^2}$, then there exists an equilibrium such that $\alpha^* = 1$. Condition (iii) imposes $2\rho\sigma^2(I_1 - I_2) \geq \delta_D - \delta_S$.[5] ■

The liquidity demander trades off the benefits of price competition in a two-venue structure (related to the divergence of inventories, $I_1 - I_2$) to the private benefits of sending the entire demand to the dominant market ($\delta_D - \delta_S$). We conclude that, even when the demand splitting is endogenized, it is still the case that the market remains ex ante fragmented.

[5]If $2\rho\sigma^2(I_1 - I_2) < \delta_D - \delta_S$, there is no solution to the FOC in$[\frac{1}{2}, 1)$. There is a corner equilibrium: $\alpha^* = 1$.

D Introduction of a pre-stage inter-dealer market

This appendix analyzes whether our results are sensitive to the introduction of a pre-stage inter-dealer market. We assume that, at stage 0, intermediaries are able to optimally share inventory risks before setting quotes in the customer-dealer market. It could be the case that they prefer sharing risks in an inter-dealer market to avoid multi-venue competition in the customer-dealer market starting at stage 1.

In a conservative approach, we assume that intermediaries independently and non-strategically maximize their expected profit in the inter-dealer market, then maximize their expected profit in the customer-dealer market (the model is solved sequentially).[6]

Even in the presence of a pre-stage risk-sharing round, intermediaries may find more profitable ex ante not to trade in the inter-dealer market as shown by the following Corollary:

Corollary D.1 *The set of parameters for which intermediaries choose not to trade in the inter-dealer market is non-empty.*

Proof. We consider two stages.

First stage: the inter-dealer market (ID). If market-maker 1 sells a quantity q at price p to market-maker 2 in the inter-dealer market, the profits of market-maker 1 and 2 respectively write:

$$\left(v_1^{ID} = \left[p - \mu - \frac{\rho\sigma^2}{2}(q - 2I_1)\right] q; v_2^{ID} = \left[\mu - \frac{\rho\sigma^2}{2}(q + 2I_2) - p\right] q\right).$$

We maximize market-makers' profits with respect to q to find market-maker 1's supply function, and market-maker 2's demand function. The crossing of the supply and demand curves yields the following symmetric equilibrium in the inter-dealer market:

$$\left(q_{ID}^* = \frac{I_1 - I_2}{2}; p_{ID}^* = \mu - \rho\sigma^2 \frac{I_1 + I_2}{2}\right).$$

At equilibrium in the inter-dealer market, market-makers' profits write $\left(v_1^{ID}\right)^* = \left(v_2^{ID}\right)^* = \frac{\rho\sigma^2}{8}(I_1 - I_2)^2$. Notice that market-makers find it optimal to perfectly share risk: after trading in the inter-dealer market, market-makers 1 and 2 end up with the same inventory position, $I_1' = I_2'$.

Second stage: the customer-dealer market (CD). Given market-makers' inventory positions after their trades in the inter-dealer market, their equilibrium profits in the customer-dealer market can be computed at the limit when $I_1' \to I_2'$ using the formula derived in the proof of

[6] In the case in which intermediaries strategically trade in the inter-dealer market *after* observing the realization of the order flows in venue D and S, we find that they may find optimal to reinforce the divergence in inventories in order to maximize their trading profit in the customer-dealer market. The inter-dealer market is not a way to optimize risk-sharing, but to enhance divergence in inventories. Multi-venue competition in the customer-dealer is thus emphasized in this case.

Proposition 1. We find: $\left(v_1^{CD|ID}\right)^* = \left(v_2^{CD|ID}\right)^* = \rho\sigma^2 q_D q_S$.

Comparison. We finally compute market-makers' expected profits in the presence of an inter-dealer market, namely $V^{CD+ID} = E\left(\left(v_i^{CD|ID}\right)^* + \left(v_i^{ID}\right)^*\right)$, and compare them with the expected profits they obtain in the absence of an inter-dealer market, namely $\left(V^{CD}\right)^* = E\left(\left(v_i^{CD}\right)^*\right)$. Computations yield:

$$V^{CD+ID} = \frac{\rho\sigma^2}{48}(I_u - I_d)^2 + \gamma\rho\sigma^2 q_D q_S, \tag{8}$$

and

$$\begin{aligned} V^{CD} &= \frac{\rho\sigma^2}{6}(I_u - I_d)\left(q_D + (2\gamma - 1)q_S\right) \\ &+ \frac{\rho\sigma^2 q_S}{(I_u - I_d)^2} \times \left[\begin{array}{l} (1-\gamma)(I_u - I_d)^3 - \left(3(1-\gamma)q_D + \frac{1}{2}\gamma q_S\right)(I_u - I_d)^2 \\ + \left\{(1-\gamma)q_D + \frac{1}{2}\gamma q_S\right\} q_D \left(3(I_u - I_d) - q_D\right) \end{array}\right]. \end{aligned} \tag{9}$$

To assess the impact of the existence of an inter-dealer market on intermediaries' expected profits, one needs to compare the expressions given in Eq. (8) and (9). Closed-form solutions are difficult to interpret. However there exist parameters' values such that intermediaries would prefer not to share risk in an inter-dealer market, that is, $V^{CD} > V^{CD+ID}$. Figure 1 shows that intermediaries are better off trading ex ante in an inter-dealer market only when (i) the probability that shocks have the same sign, γ, is high, and (ii) the size of the liquidity demand sent to the satellite venue, q_S, is small.

As illustrated by Figure 1, there exist cases (white squared surface) in which intermediaries find more profitable ex ante not to trade in the inter-dealer market (for different values of γ and q_S) and trade directly in the customer-dealer market.■

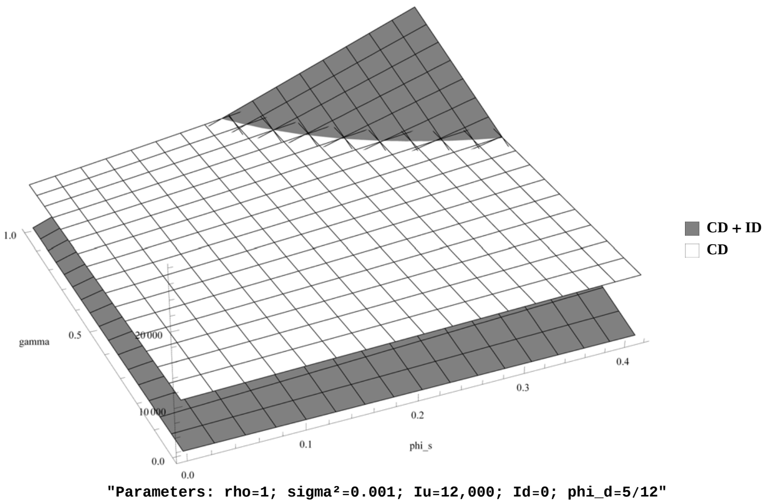

Figure 1: Impact of the inter-dealer market on dealers' expected profits.

Figure 1 represents intermediaries' expected profits with or without an initial trading round in an inter-dealer market, as a function of γ (the probability that shocks have the same sign) and ϕ_S, for $\phi_S \leq \phi_D$ and $\phi_D \leq I_u - I_d$. The white squared surface plots the expected trading profit in the customer-dealer market (CD) only, the grey squared surface plots the total expected trading profit if intermediaries engage in an inter-dealer round before trading in the customer-dealer market (CD+ID).

E No trade-through

This section explores whether a market order can execute at a price worse than the best quoted price, termed as trade-through. Note that a trade-through can only occur if orders sent to S and D have the same sign. The question is thus: do we observe different prices across trading venues when orders Q_D and Q_S are of same direction and same size?

Corollary E.1 *There is no trade-through possible in our model.*

Proof: We use Proposition 1 when Q_D and Q_S have the same sign, and consider that $Q_S = Q_D = Q$.

- If $(I_1 - I_2 - Q_D)Q_S > 0$ and $Q_D = Q_S = Q > 0$, then it is straightforward to show that $(a_1^D)^* = (a_1^S)^* = r_2(Q)$.

- When $(I_1 - I_2 - Q_D)Q_S \leq 0$ and $Q_D = Q_S = Q > 0$, easy computations (below) show that $(a_1^D)^* = (a_2^S)^*$.

Observe that $\hat{r}_2(Q) - \rho\sigma^2 Q \times \eta = r_2(Q) + \rho\sigma^2 Q - \rho\sigma^2(I_1 - I_2) = r_1(Q) + \rho\sigma^2 Q = \hat{r}_1(Q)$. We deduce that $(a_1^D)^* = (a_2^S)^*$. ■

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